Approximate strong edge-colouring of unit disk graphs

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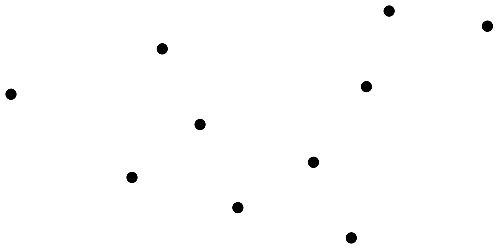
March 2019

A telecommunication problem

Unit disk graphs [Clark, Colbourn, Johnson 1990]

- Transceivers communicate iff Euclidean distance ≤ 1
- Undirected graph: transceivers send and acknowledge



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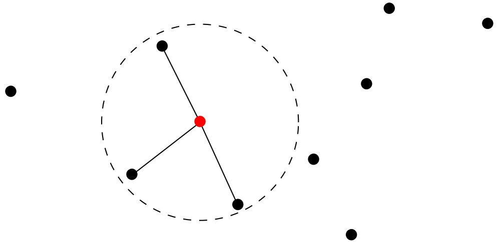

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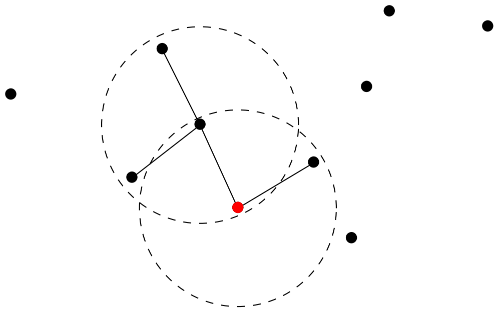

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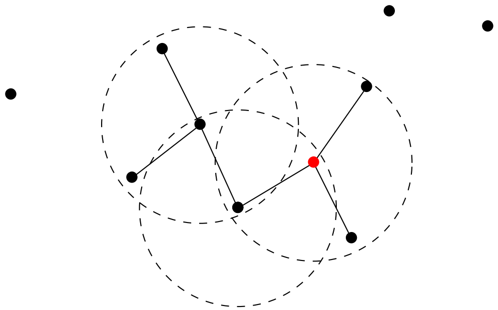

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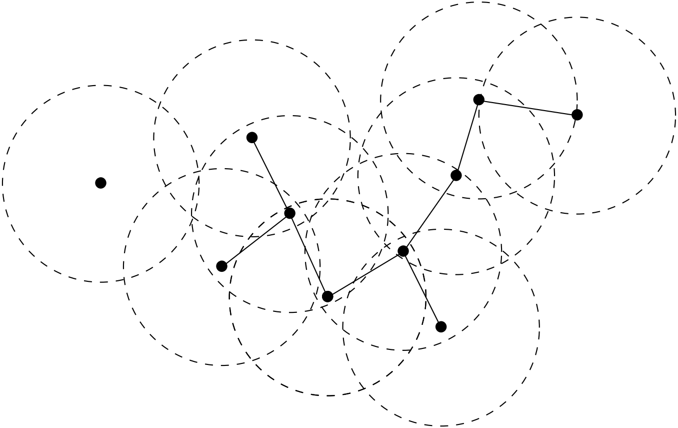

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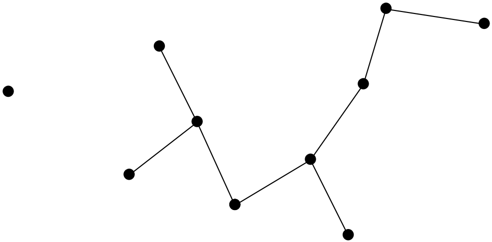
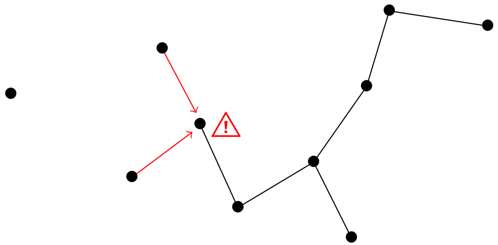


Channel assignment in radio networks

- Assign frequencies to pairs of communicating transceivers
- Avoid primary and secondary interferences
- Minimise the range of frequencies used



Primary interference

Channel assignment in radio networks

- Assign frequencies to pairs of communicating transceivers
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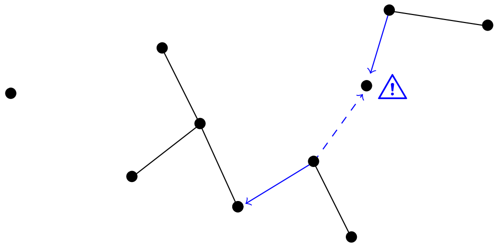
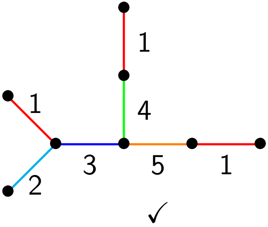


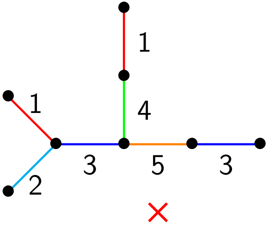
Secondary interference

Interference model

Strong edge colouring [Ramanathan, Lloyd 1993] [Barrett et al. 2006]

- Assumption: frequencies are discrete
- Two edges at distance 1 or 2 are assigned different colours, i.e. each colour set induces a matching
- Strong chromatic index = minimum number of colours needed





Results

Theorem

Finding the strong chromatic index of UDGs is a NP-hard problem

→ want approximation algorithms

	Previous ratio	New ratio
Offline algorithm	8 [1]	6
Online algorithm	10 [2]	8

[1] Barrett et al. (2006)

[2] Kanj, Wiese and Zhang (2011)

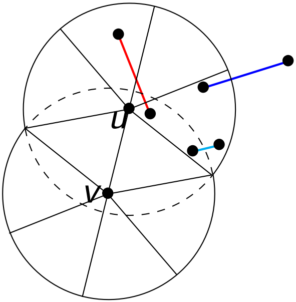
10-approximation of the strong chromatic index

Lemma

If e, e' have a vertex in the same section → different colours

Theorem

If $\{u, v\}$ receives colour $k+1$ → need at least $k/10$ colours

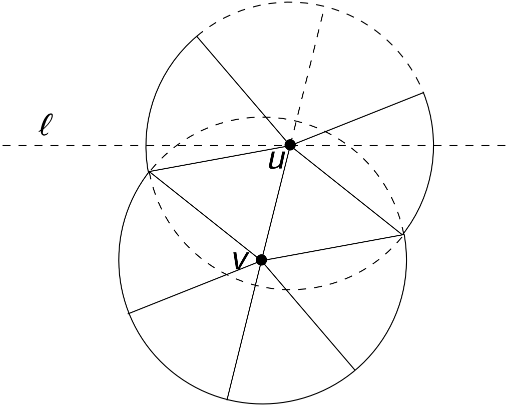


10 sections with unit diameter

Kanj, Wiese and Zhang (2011)

8-approximation of the strong chromatic index

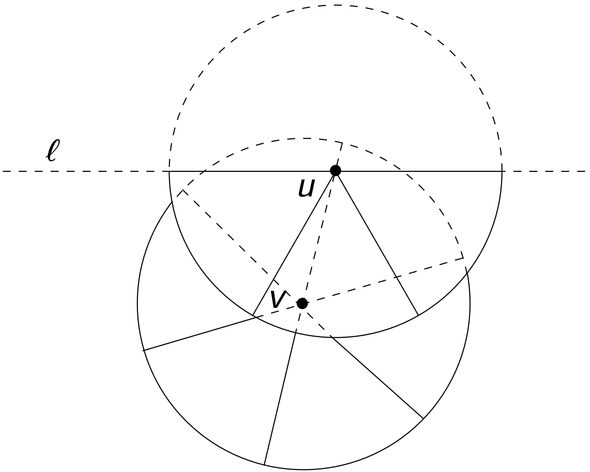
Order edges by the highest vertex → edges already coloured are below ℓ



8 sections with unit diameter

Barrett et al. (2006)

7-approximation of the strong chromatic index



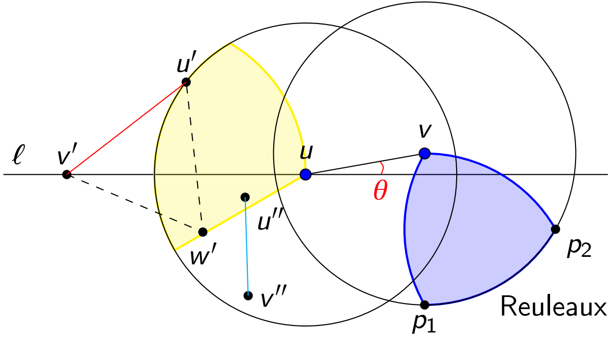
7 sections with unit diameter

6-Approximation of the strong chromatic index

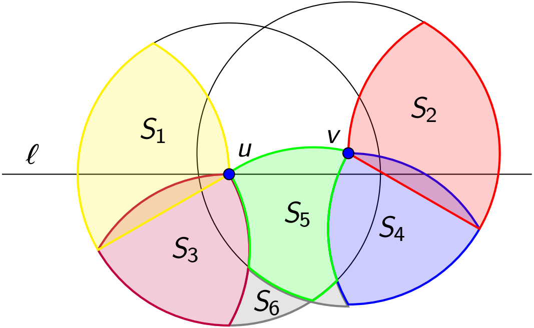
- Reuleaux triangles,
- New ordering: consider the lowest vertex,
- Small sections

Lemma

If e', e'' have a vertex in a small section → different colours

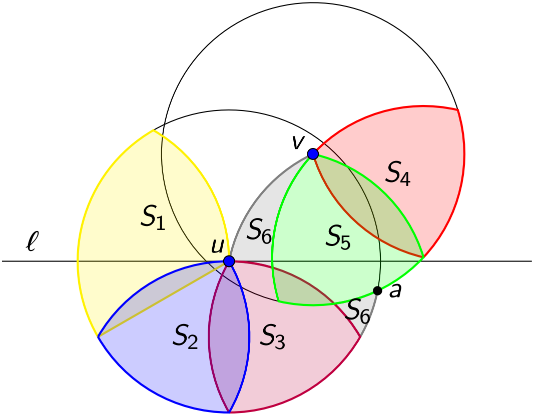


6-approximation of the strong chromatic index



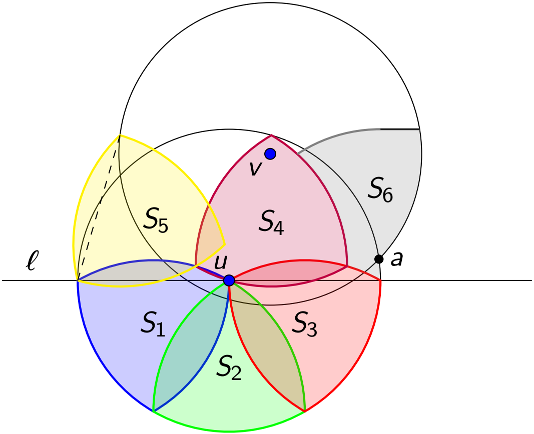
The six sections when $0 \leq \theta < \pi/6$.

6-approximation of the strong chromatic index



The six sections when $\pi/6 < \theta$ and a is below ℓ.

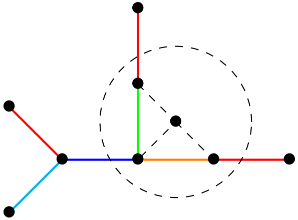
6-approximation of the strong chromatic index



The six sections when a is above ℓ.

Online 8-approximation of the strong chromatic index

New transceiver → assign frequencies to new channels



First Fit algorithm: assign the smallest available colour

Theorem

First Fit algorithm gives an 8-approximation

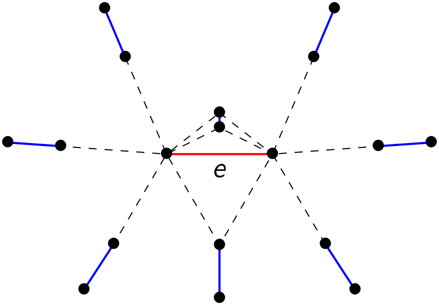
Online 8-approximation of the strong chromatic index

Theorem

Cannot have 9 *independent edges in* $2\text{-}N(e)$

Corollary

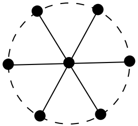
Cardinality of each colour set in $2\text{-}N(e)$ *is* ≤ 8



8 independent edges

Online 8-approximation of the strong chromatic index

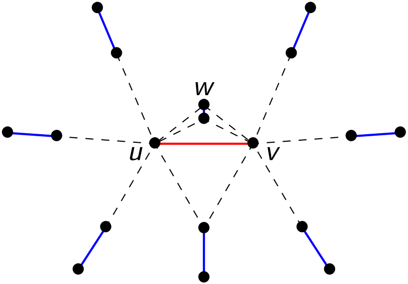
Star graph S_6 is not a unit disk graph



Consider one vertex for each edge

Lemma

$\{w_i\}_{1 \le i \le 8} \subset \mathcal{N}(u) \cup \mathcal{N}(v)$ *pairwise independent*
$\implies \exists w_i \in \mathcal{N}(u) \cap \mathcal{N}(v)$



Online 8-approximation of the strong chromatic index

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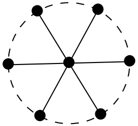

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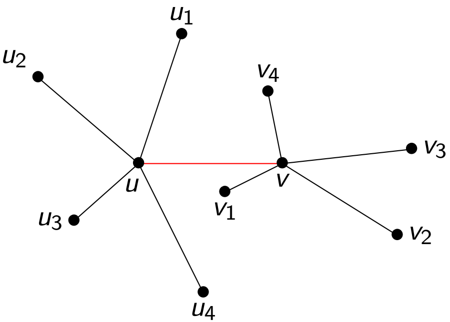
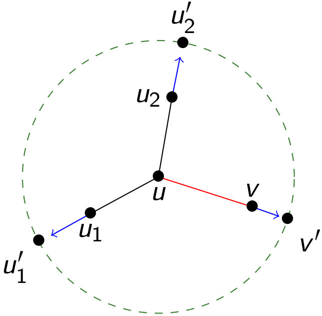


$\{u, v\} \in E$, $\{u, u_i\} \in E$, $\{v, v_j\} \in E$

Shifting vertices to the boundary

Consider vertices in $N(u)$ only

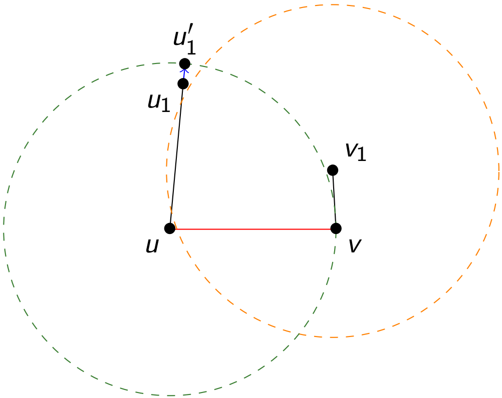
u_1, u_2, v pairwise independent → u_1', u_2', v' pairwise independent



Shifting at least one vertex to the boundary

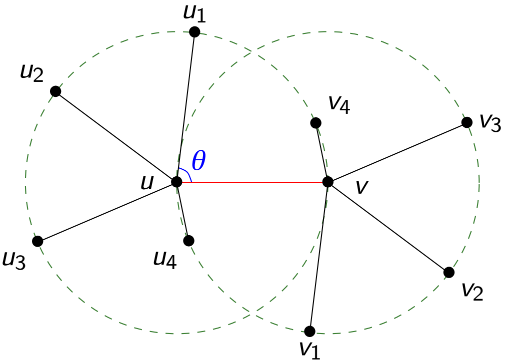
Take $u_1 \in N(u)$ and $v_1 \in N(v)$

u_1, v_1 independent $\rightarrow$ u_1', v_1 independent



Proof of the lemma

- Shift as many vertices as possible
- $\sphericalangle vuu_i \geq \theta + (i-1)\pi/3$
  $\sphericalangle uvv_3 \geq \theta + \pi/3$
- Contradiction: $\sphericalangle uvv_1 \geq \theta + \pi \rightarrow \{u_4, v\} \in E$



Conclusion

Theorem

No approximation ratio smaller than $7/6$ *unless P=NP*

Theorem

Offline 6*-approximation and online* 8*-approximation*

- May need to rely on the structure of the graphs for better approximations.