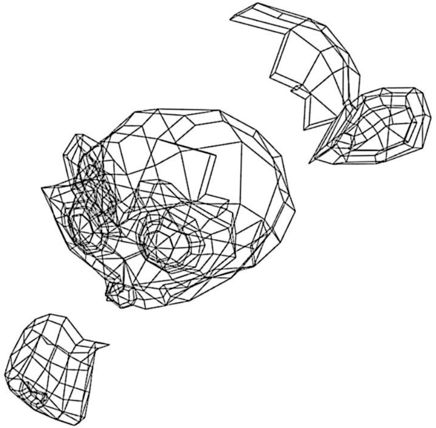
RHARBAOUI Wassim – ALAYRANGUES Sylvie – LIENHARDT Pascal – PELTIER Samuel

LOCAL COMPUTATION OF HOMOLOGY VARIATIONS RELATED TO CELLS MERGING

Theoretical results and implementation issues

1: CONTEXT AND OBJECTIVES

1 CONTEXT AND OBJECTIVES



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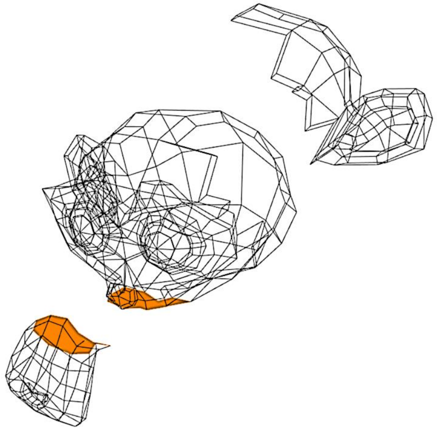

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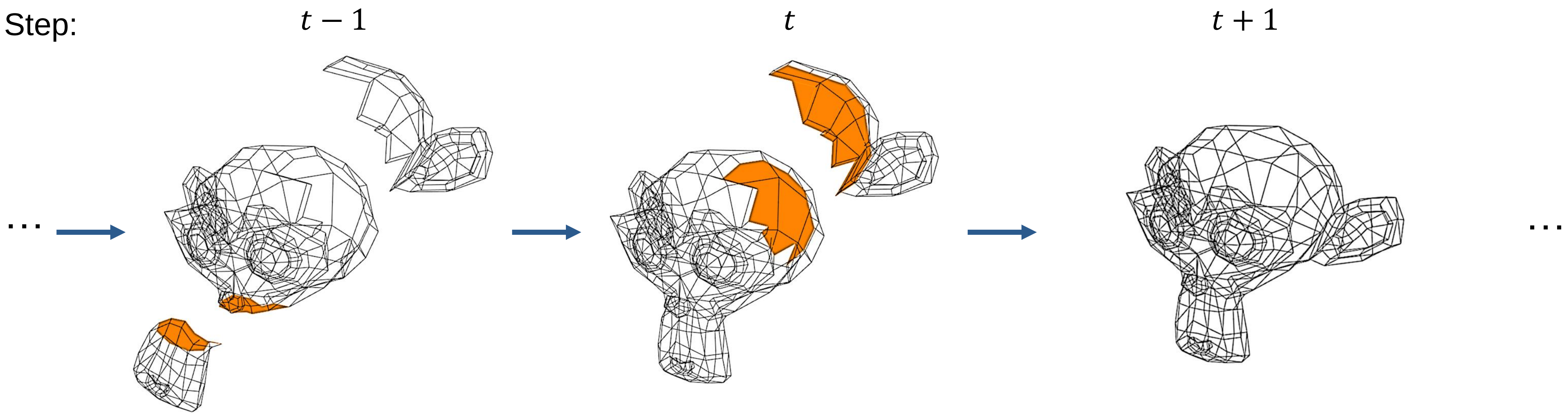
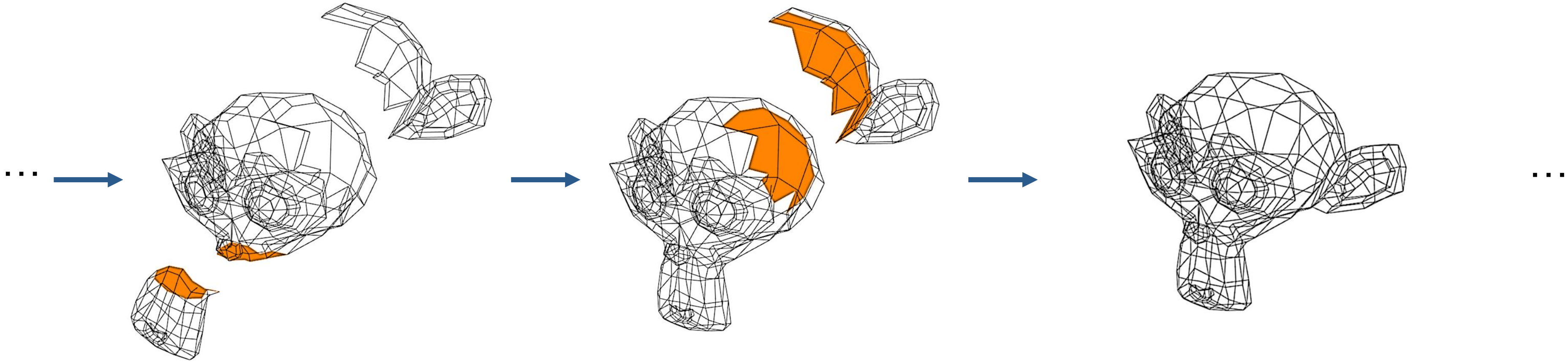


1 CONTEXT AND OBJECTIVES

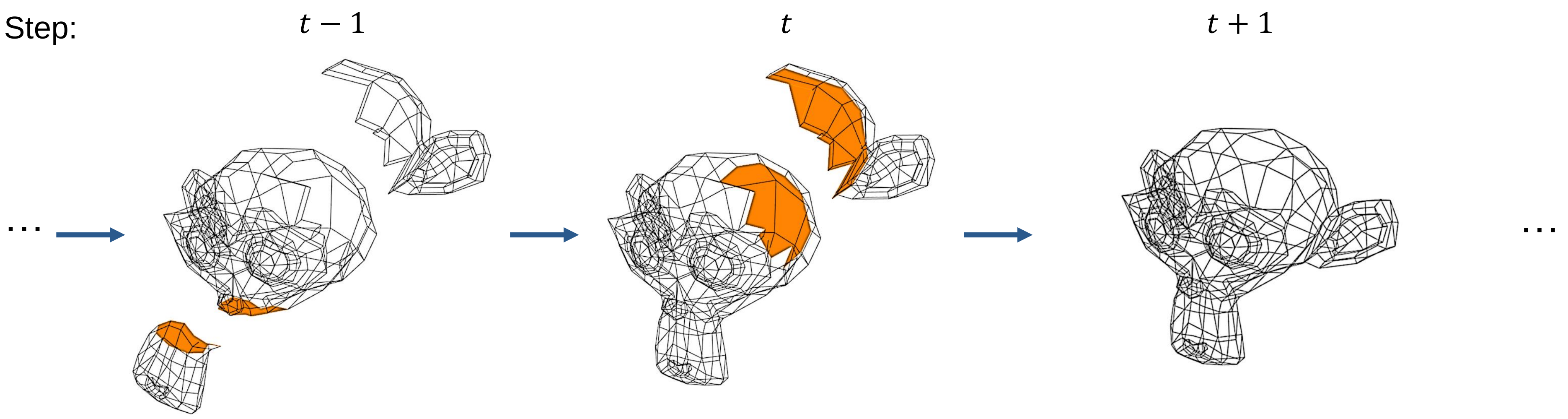
Step: $t-1$ t $t+1$



Operation: merging cells

1

CONTEXT AND OBJECTIVES



Operation: merging cells

At each step: control of the object construction according to computed information

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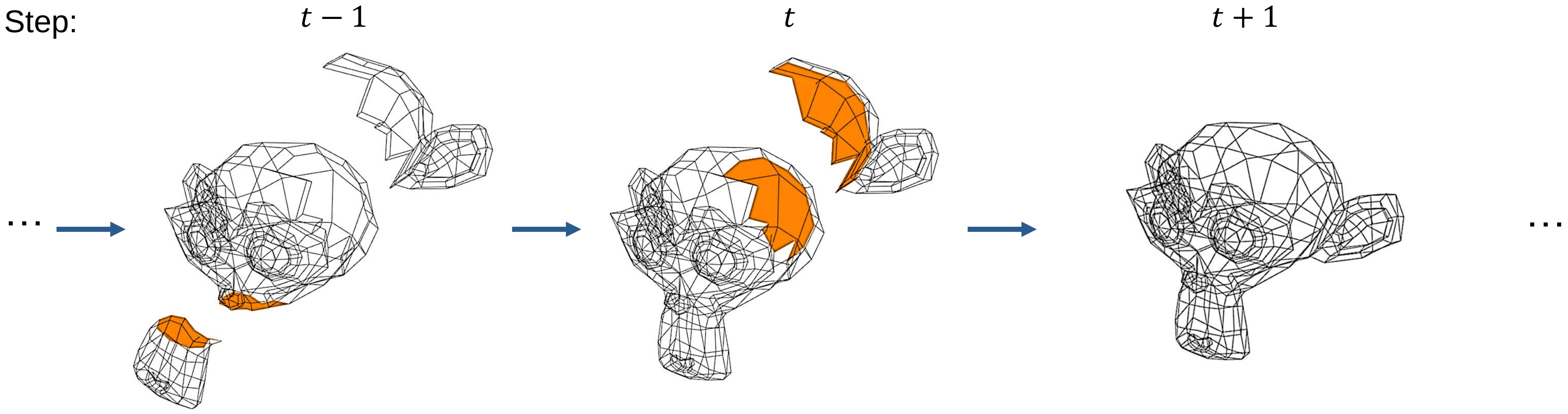


Operation: merging cells

At each step: control of the object construction according to computed information

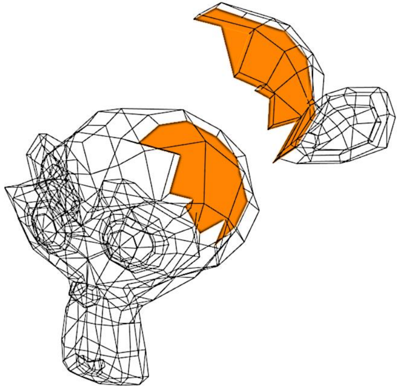
=> Compute homology efficiently

2: STATE OF THE ART

2

SMITH NORMAL FORM (SNF)

t



... → → ...

At each step:

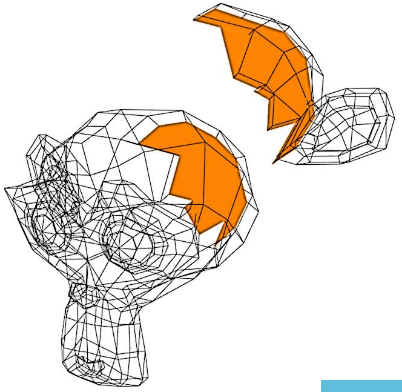
Boundary matrices

- Munkres, James R. (1984). *Elements of algebraic topology*.
- Hatcher, Allen. (2002). *Algebraic topology.*

2

SMITH NORMAL FORM (SNF)

t

… →  → …

At each step:

Boundary matrices → SNF Matrices

Smith Normal Form

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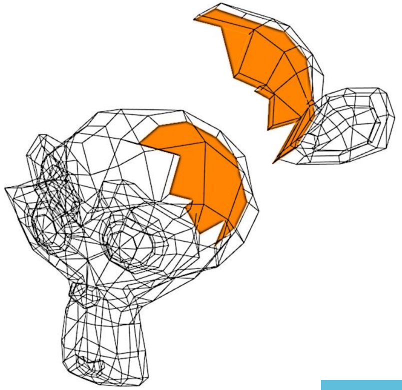
… →  → …

At each step:

Boundary matrices → (Smith Normal Form) → SNF Matrices → (Retrieving homology) → Homology groups + Generators

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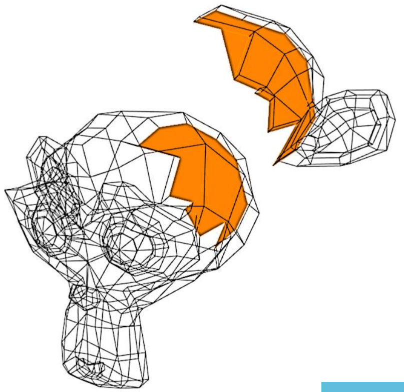
SMITH NORMAL FORM (SNF)

COMPLEXITY: SIZE OF THE OBJECT

t

… →  → …

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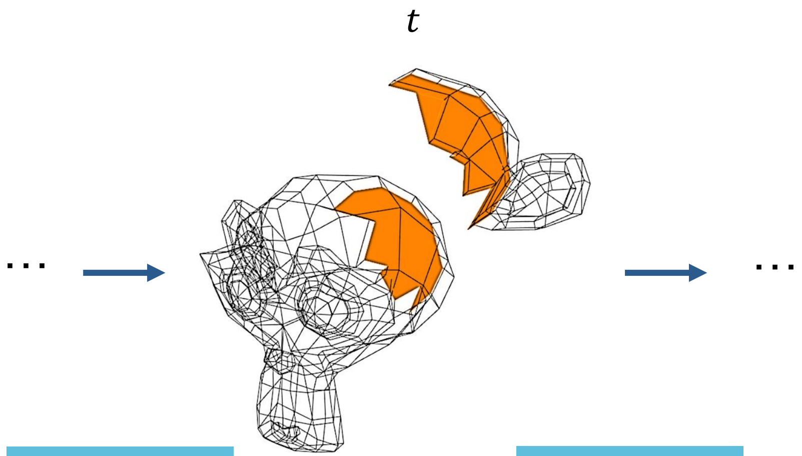
2 REDUCTION



At each step:

Boundary matrices → (Reduction) → Reduced matrices → (Smith Normal Form) → SNF Matrices → (Retrieving homology) → Homology groups + Generators

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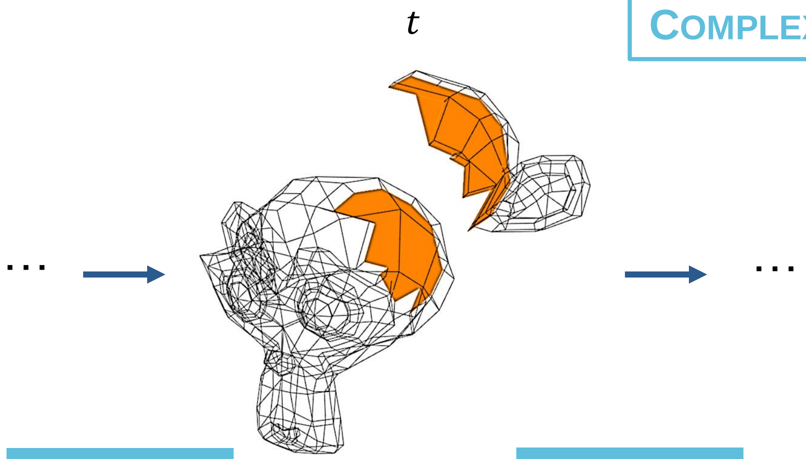

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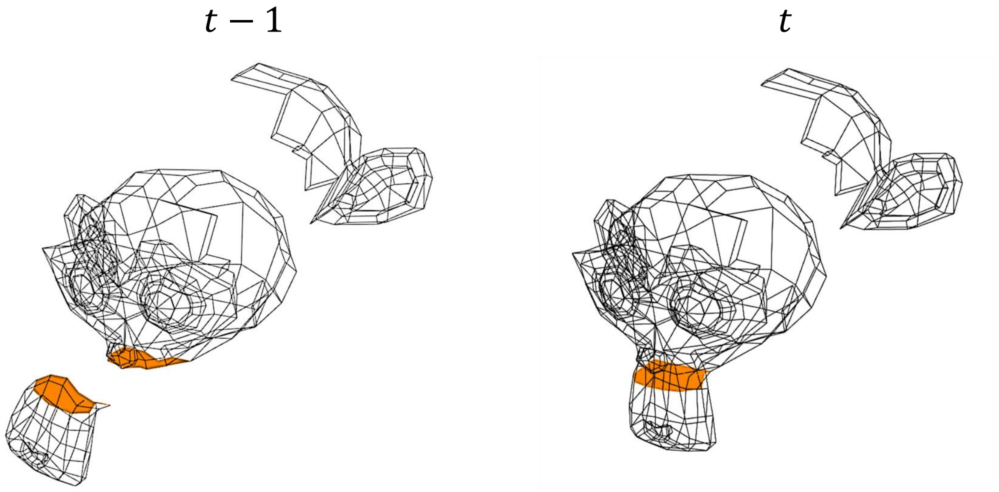
PERSISTENT HOMOLOGY



- Zomorodian, A., & Carlsson, G. (2005). *Computing persistent homology.*
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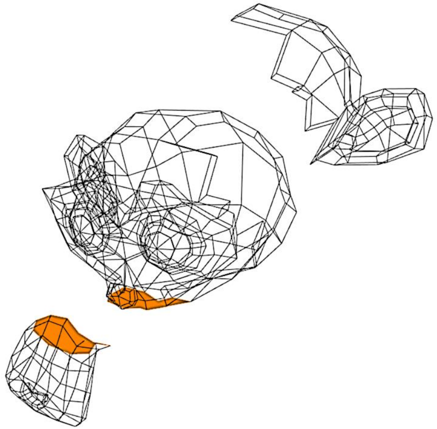
2

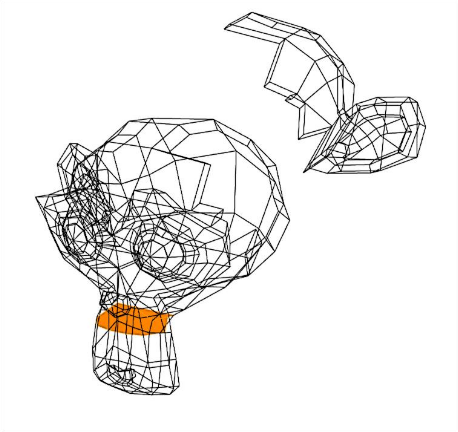
PERSISTENT HOMOLOGY

NO FILTRATION WHEN MERGING CELLS

$t-1$



t



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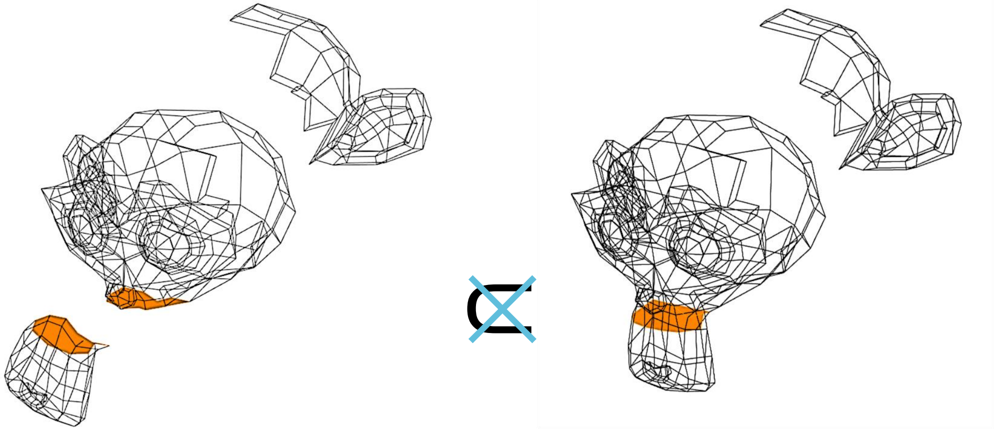

- From $\boldsymbol{t-1}$ to $\boldsymbol{t}$: merged cells

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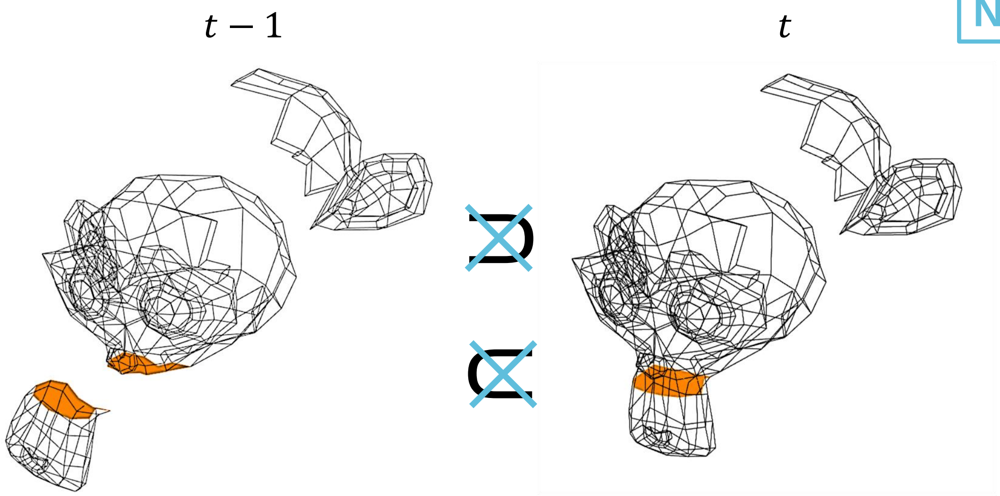

- From $\boldsymbol{t-1}$ to $\boldsymbol{t}$: merged cells
- From $\boldsymbol{t}$ to $\boldsymbol{t-1}$: modified boundary

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SHORT EXACT SEQUENCE BASED METHOD

Goal

- Tracking homology variations induced by a merging operation

- D. Boltcheva, D. Canino, S. Merino Aceituno, J.-C. Leon, L. De Floriani, and F. Hetroy. An iterative algorithm for homology computation on simplical shapes.
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- Taking advantage of locality => complexity: size of the operated part

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Contributions

- Theoretical and experimental complexity in the case of the merging cells operation

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2 SHORT EXACT SEQUENCE BASED METHOD

Goal

- Tracking homology variations induced by a merging operation
- Taking advantage of locality => complexity: size of the operated part

Contributions

- Theoretical and experimental complexity in the case of the merging cells operation
- Highlighting critical cases

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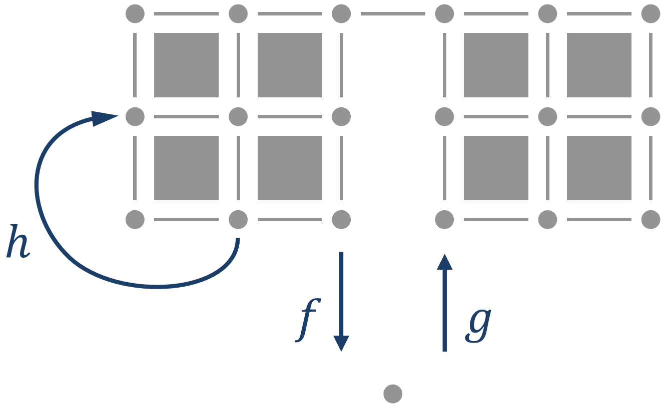
3: TOOLS

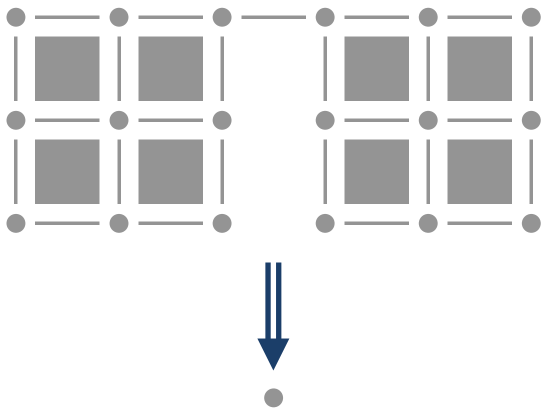
3 REDUCTION



Reducing object while preserving homology

- 3 morphisms h, f, g

REDUCTION



Object

Reduced object

Reducing object while preserving homology

- 3 morphisms h, f, g

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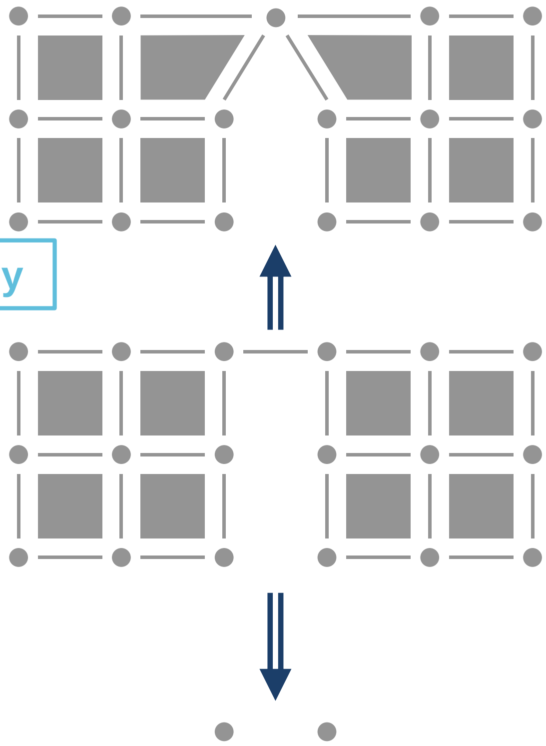
3

HOMOLOGICAL EQUIVALENCE



3 objects with same homology

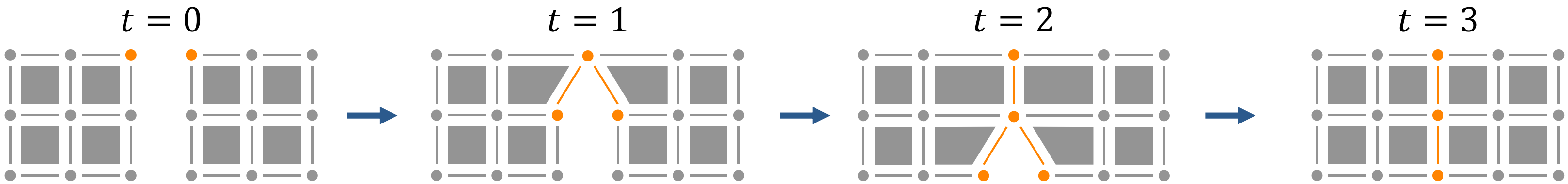
- 2 reductions

4: METHOD

4

METHOD: STUDIED OPERATION SET

3 operations, 4 construction steps

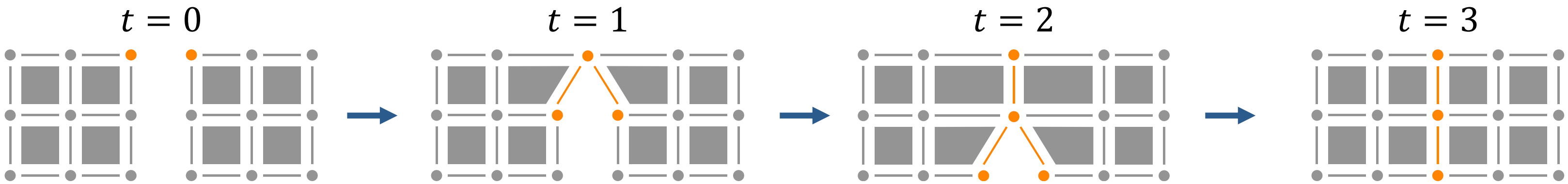


4

METHOD: PRINCIPLE

3 operations, 4 construction steps

Maintain a homological equivalence



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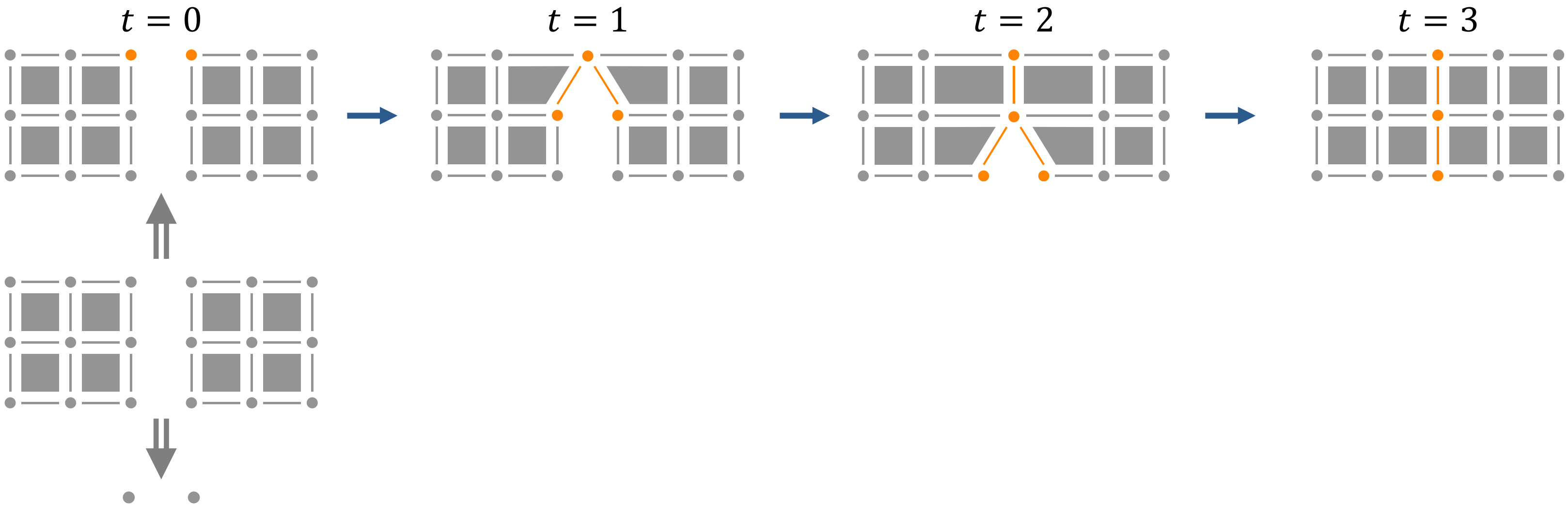

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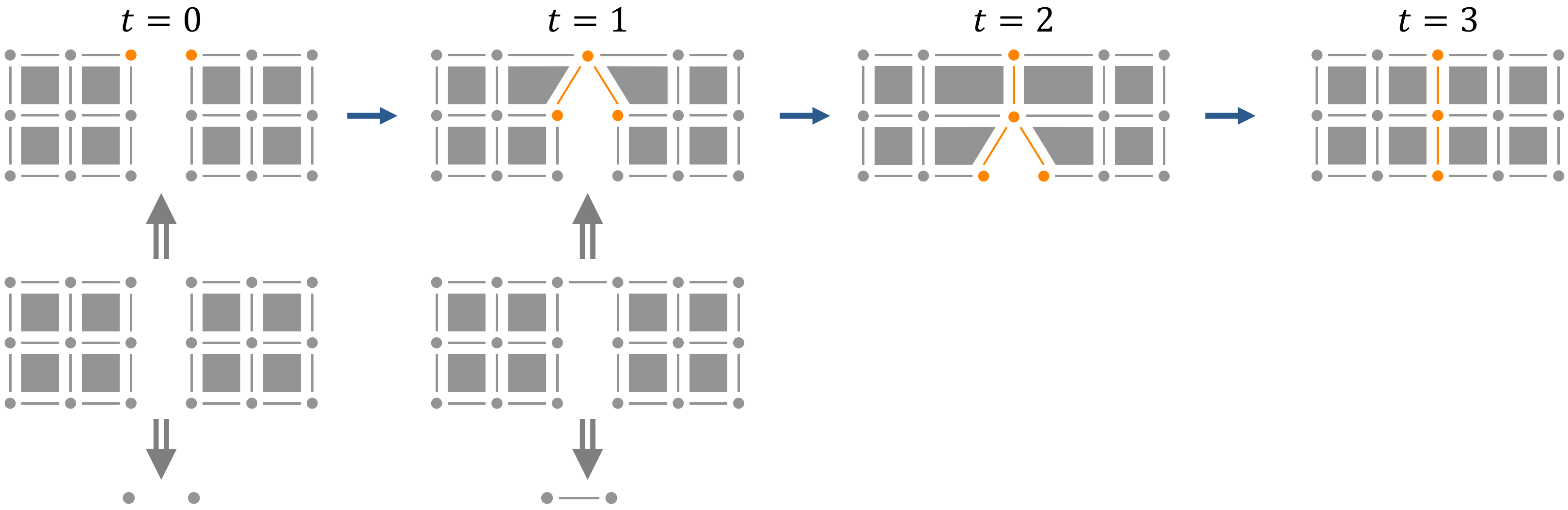

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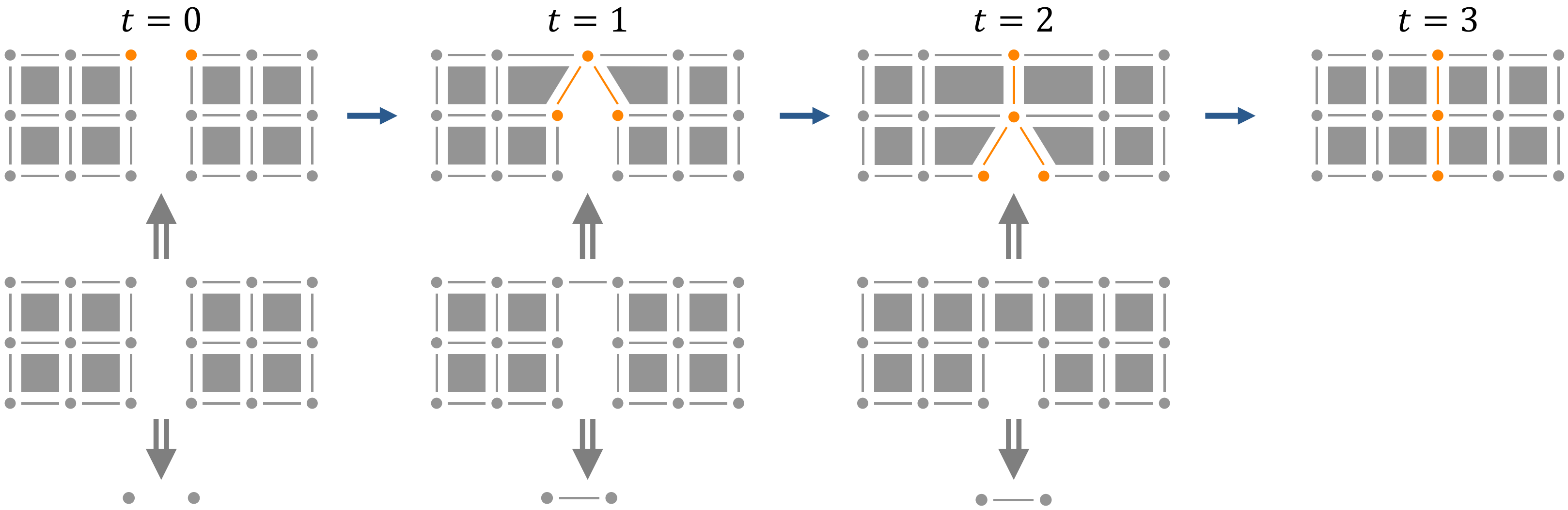

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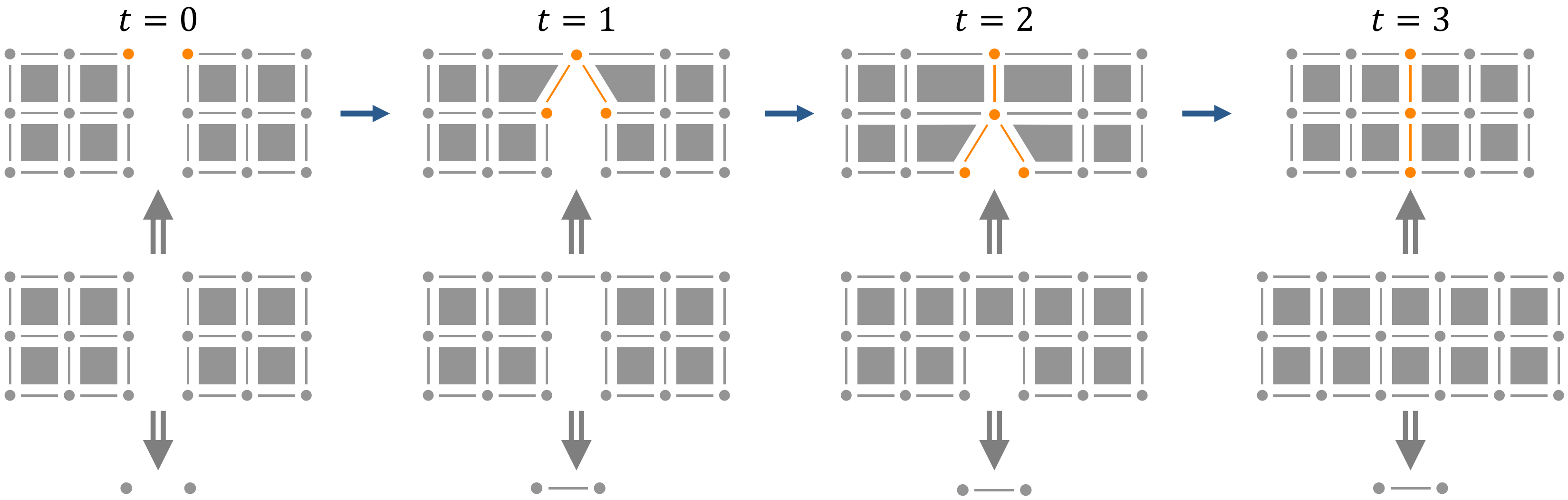

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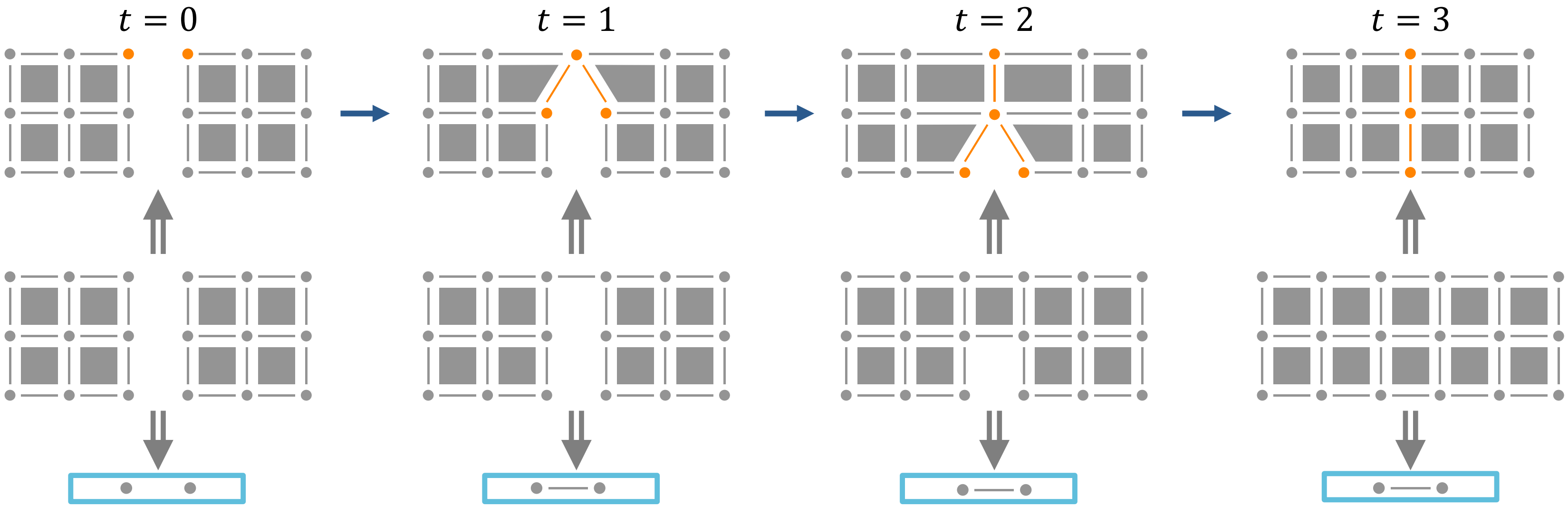
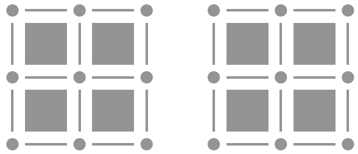


At each step : compute homology on reduced objects

4

METHOD: INITIALIZATION

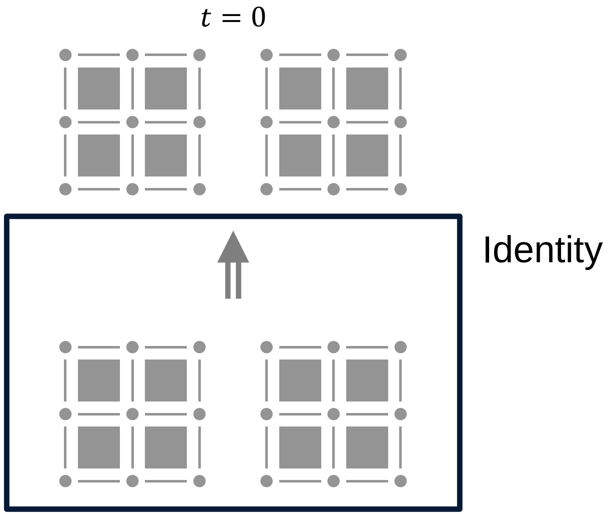
$t = 0$



Initial object

4 METHOD: INITIALIZATION

Initial object => build a homological equivalence



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METHOD: INITIALIZATION

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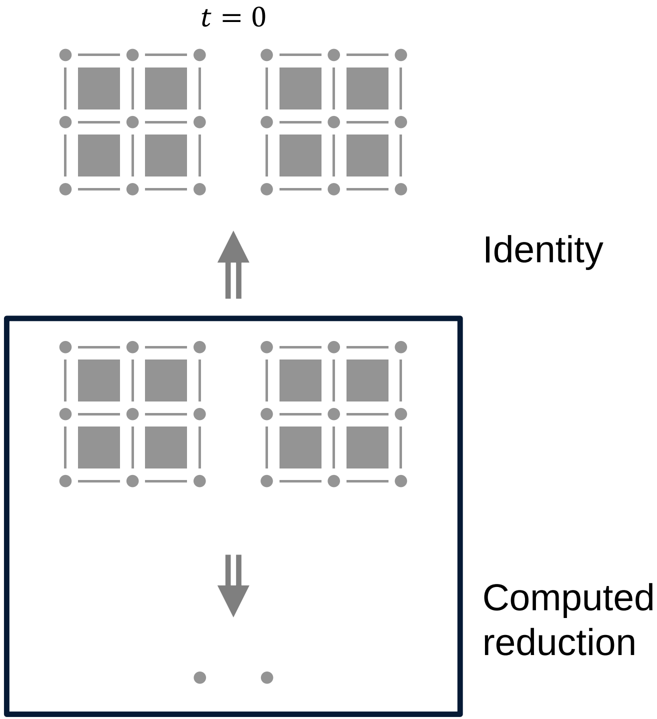

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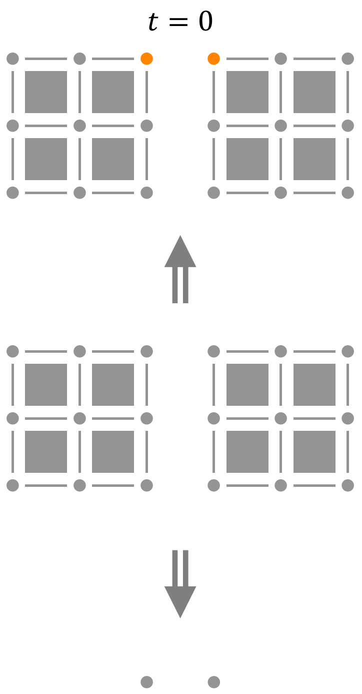
4 METHOD: WHEN MERGING CELLS



- ❑ Operation
- ❑ Initialization

4

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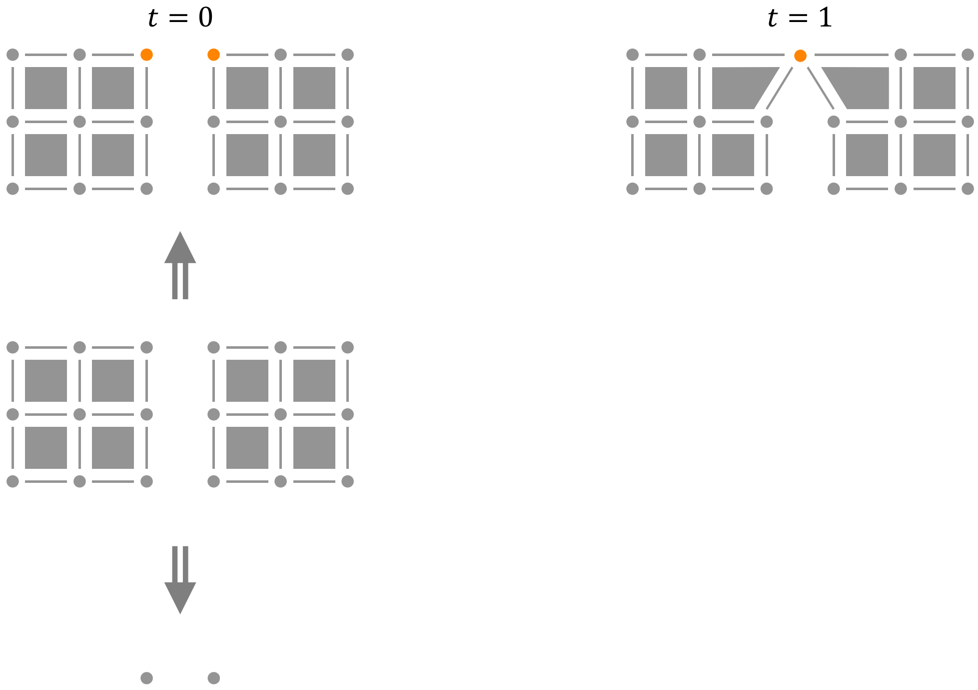

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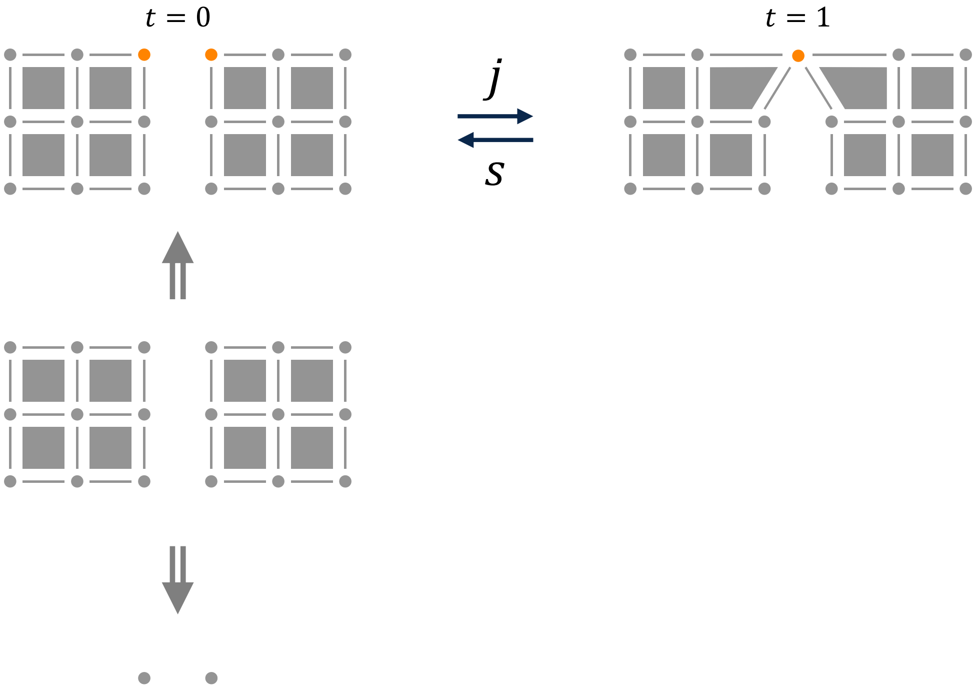

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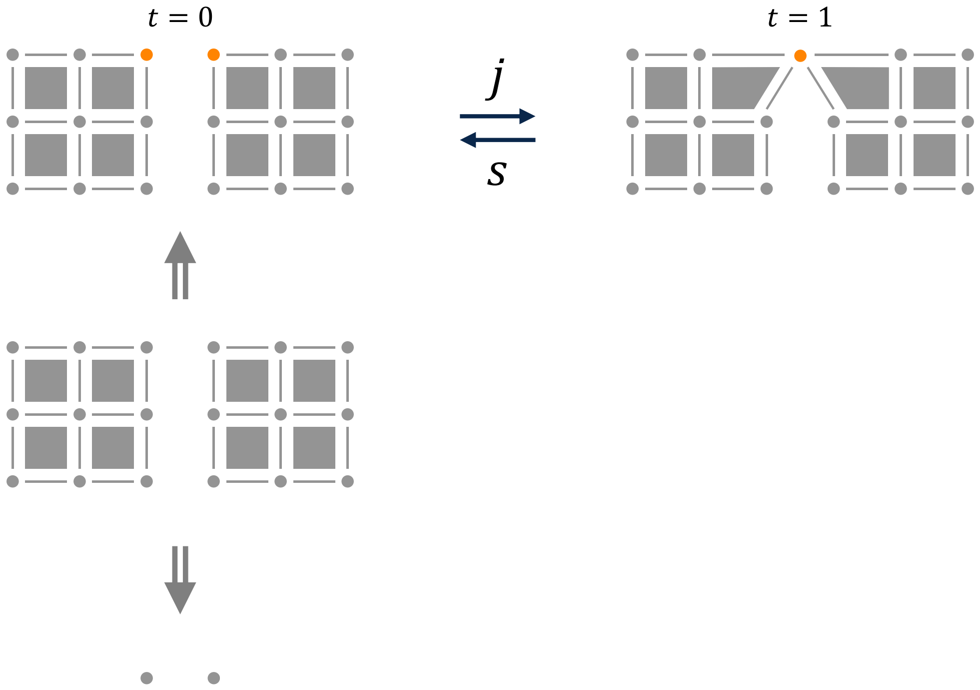

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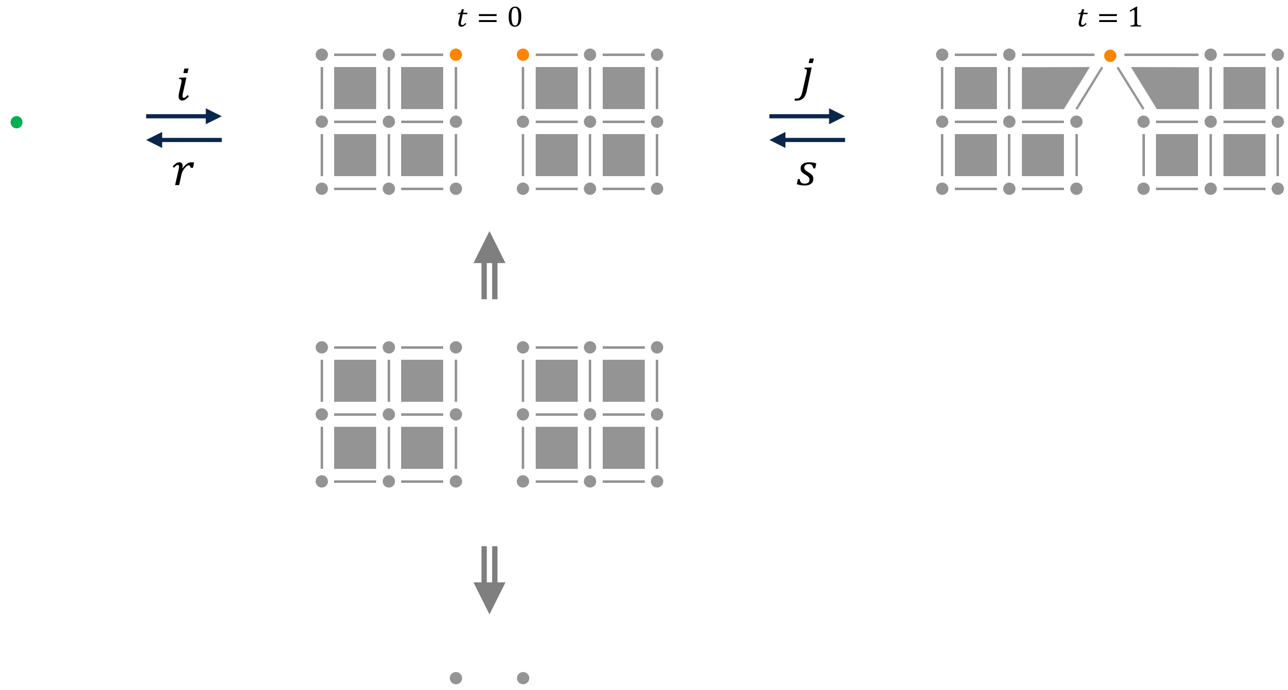

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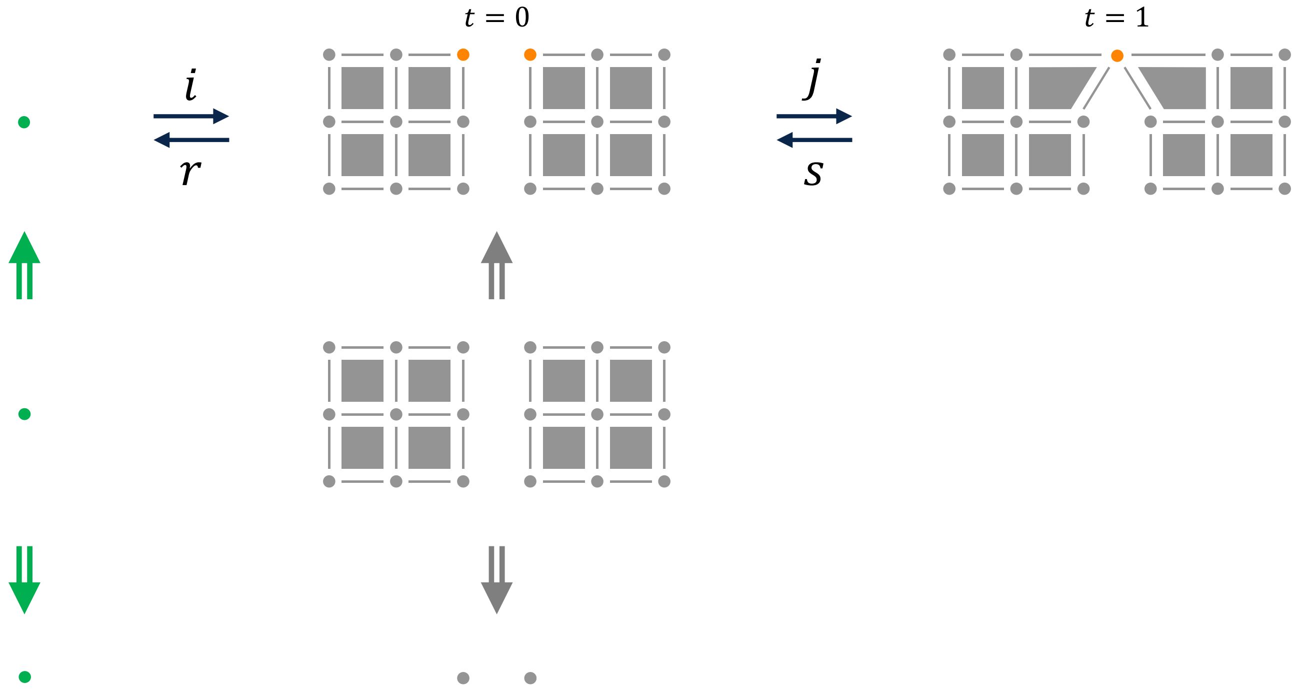

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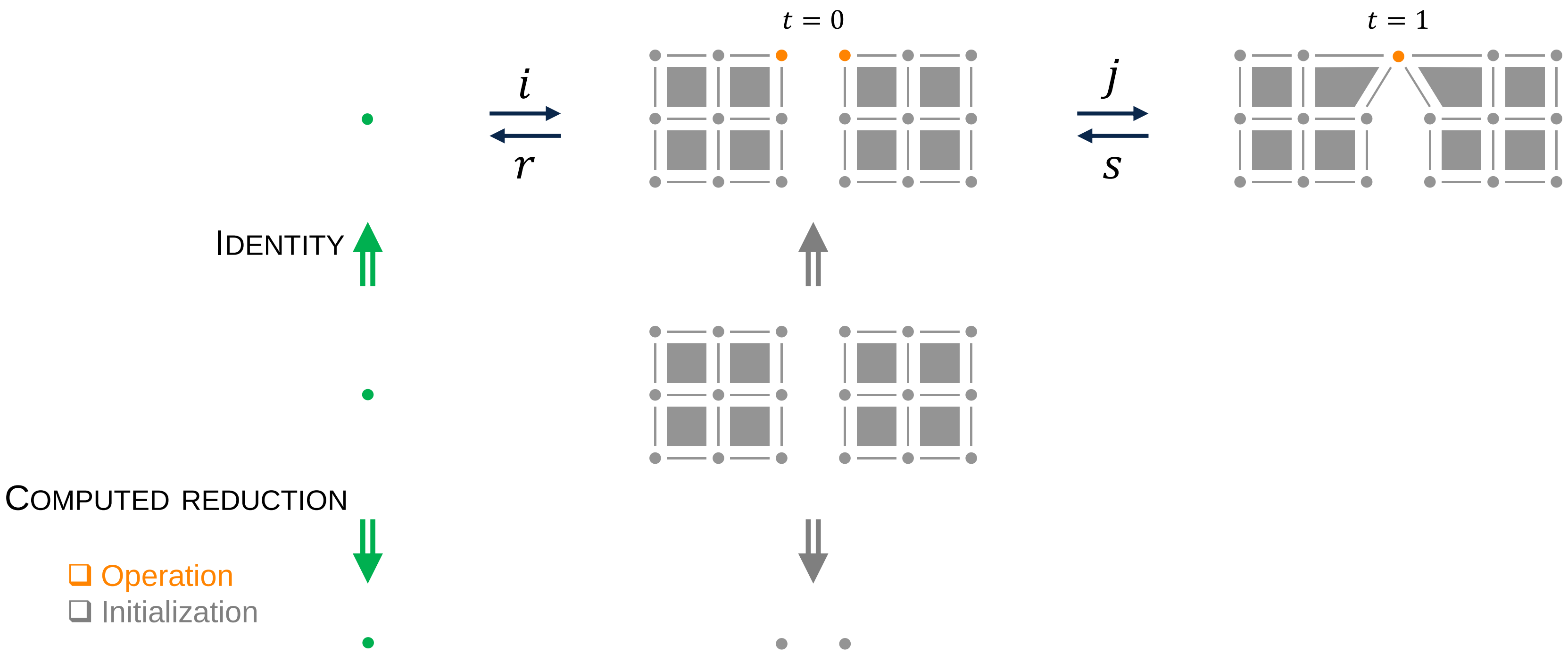

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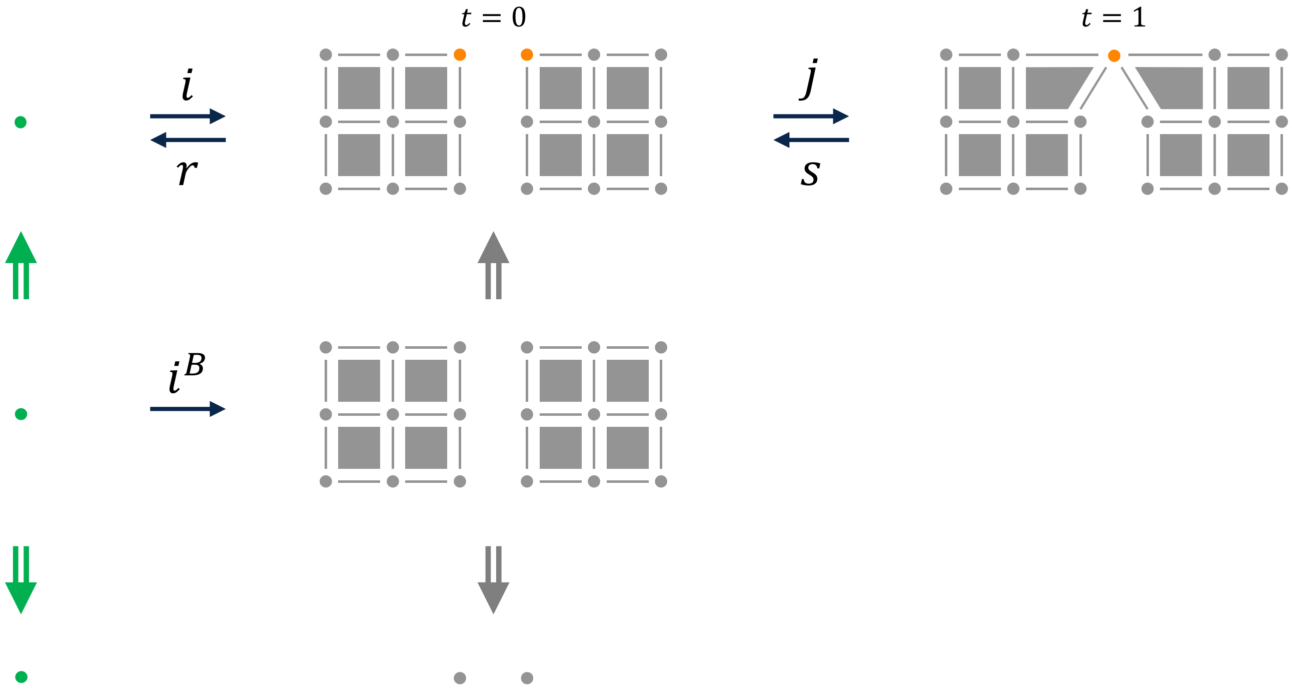

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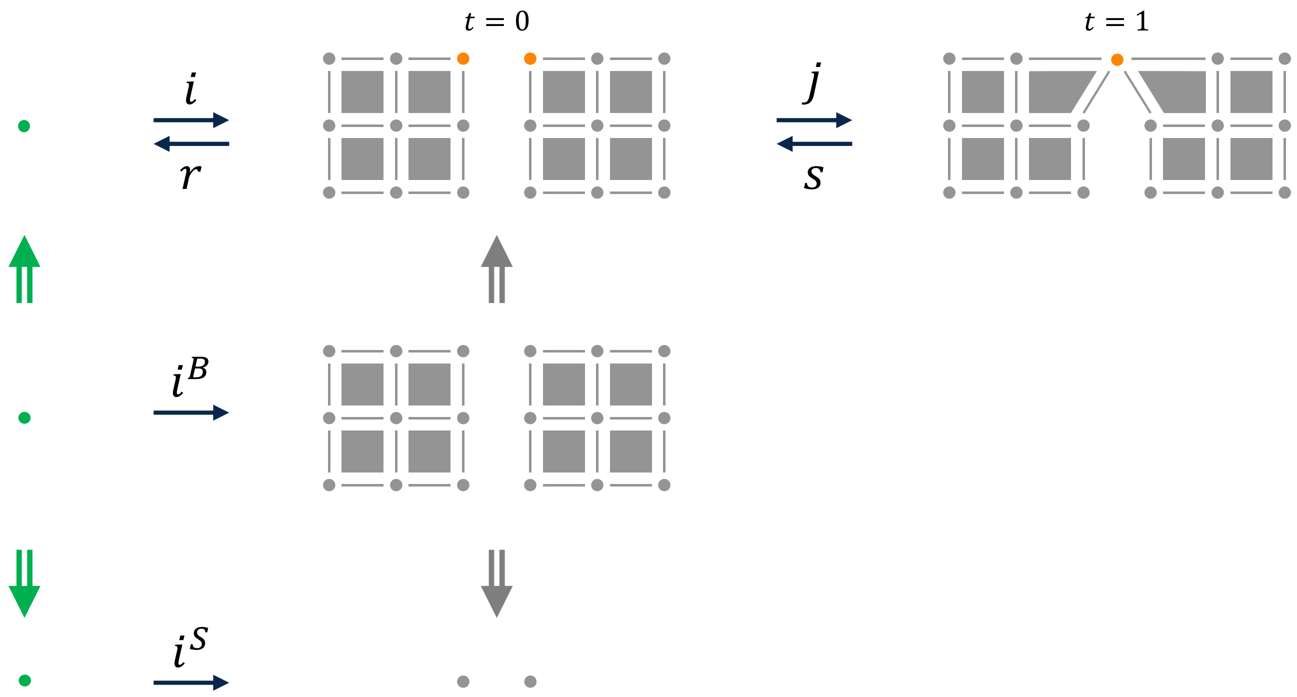

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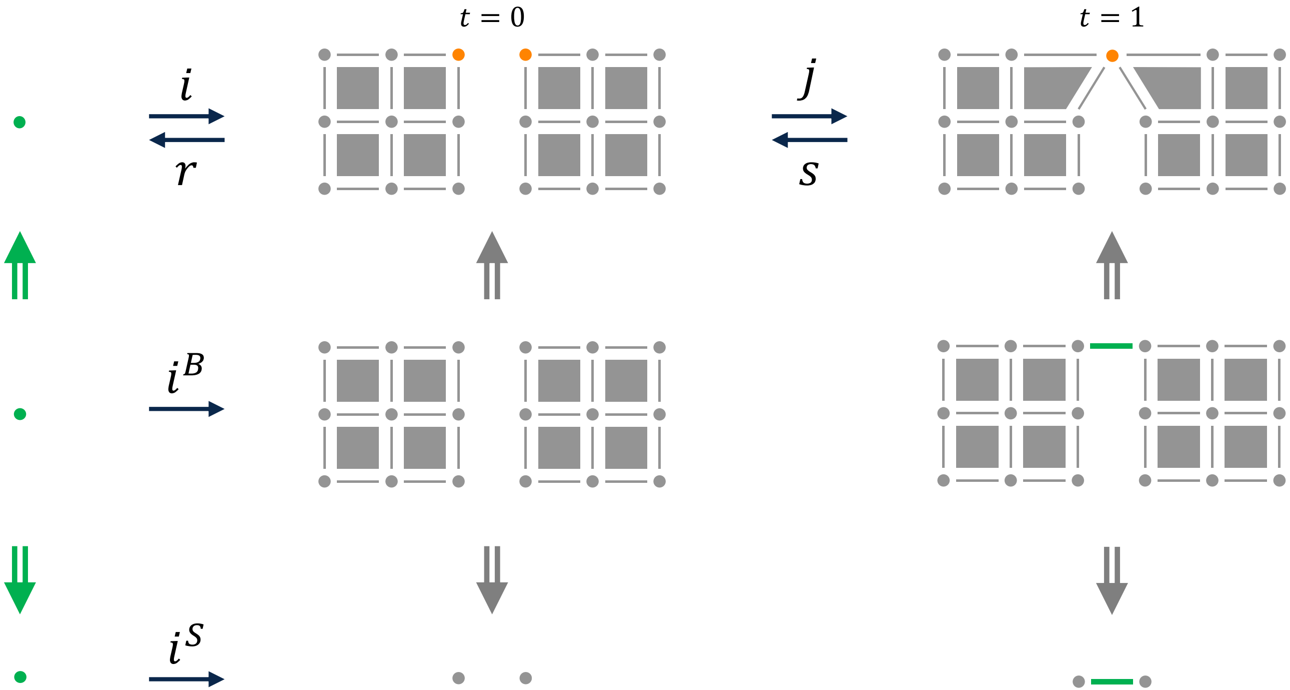
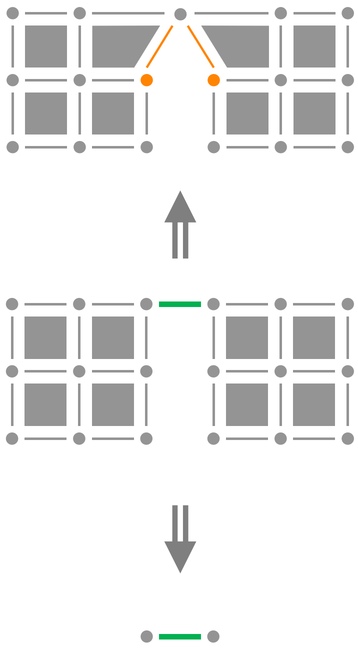


❑ Operation
❑ Initialization

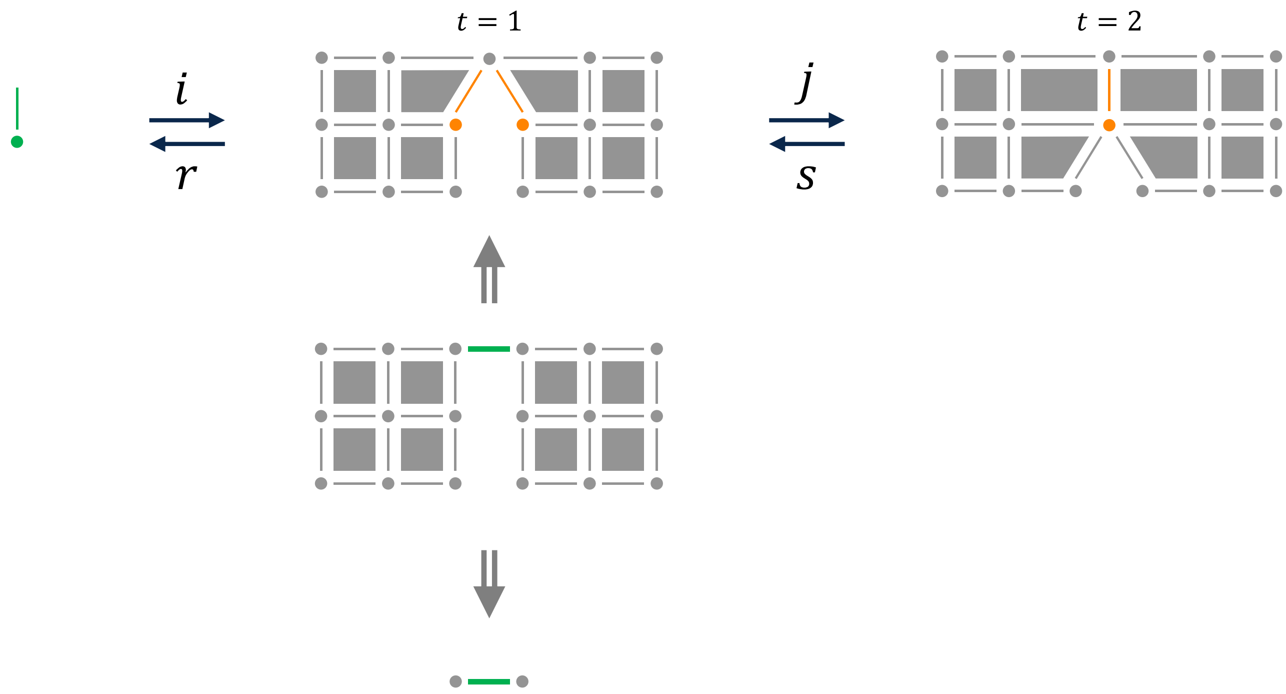
METHOD: WHEN MERGING CELLS

$t = 1$



- ❑ Operation
- ❑ Added complexity
- ❑ Initialization

METHOD: WHEN MERGING CELLS



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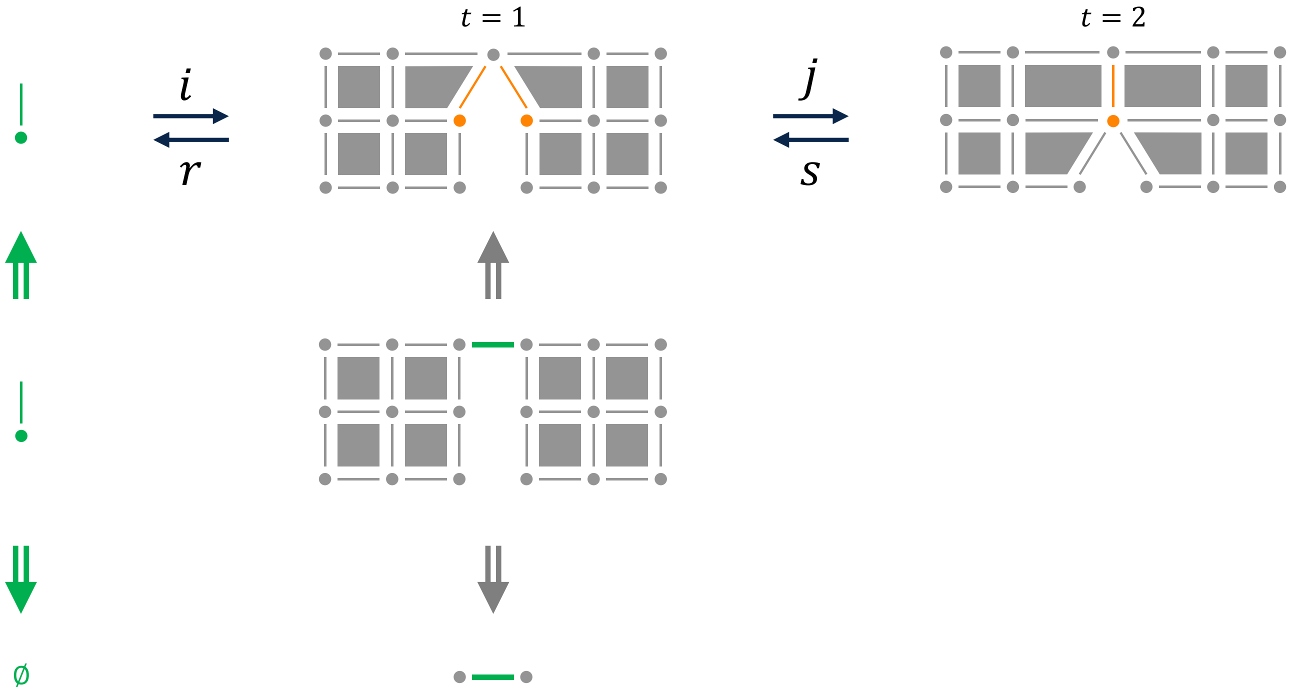

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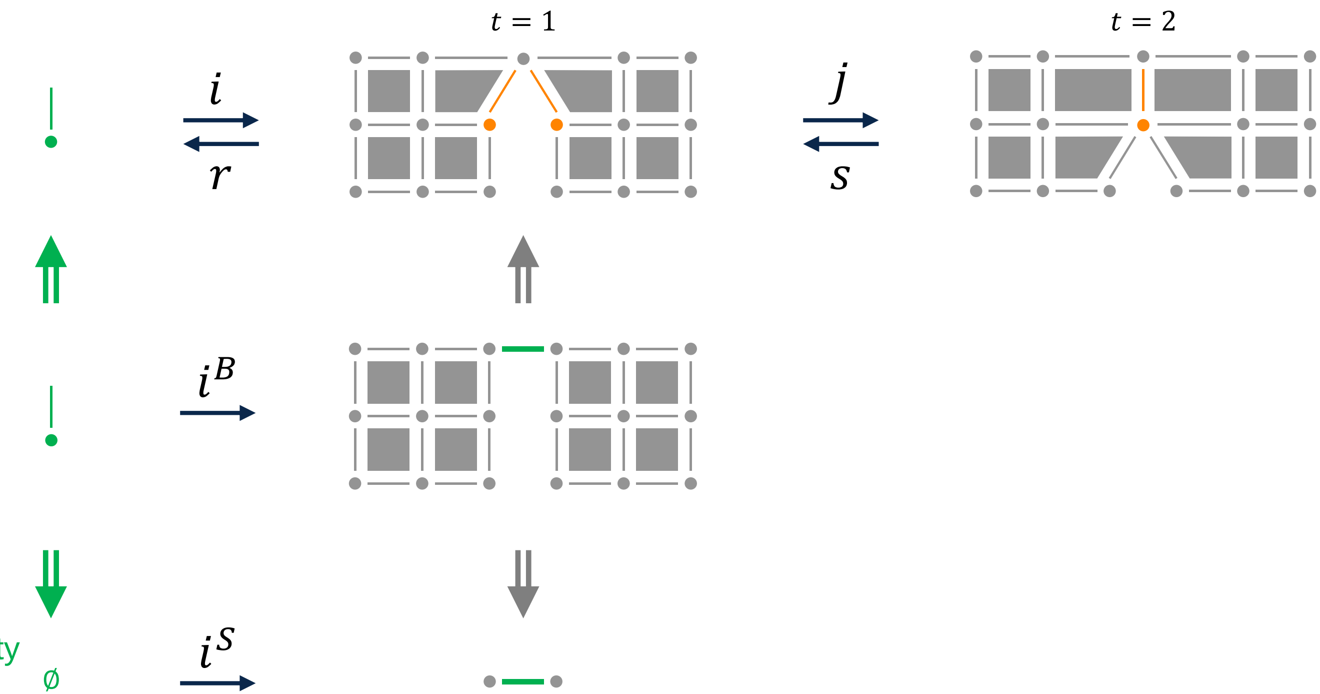

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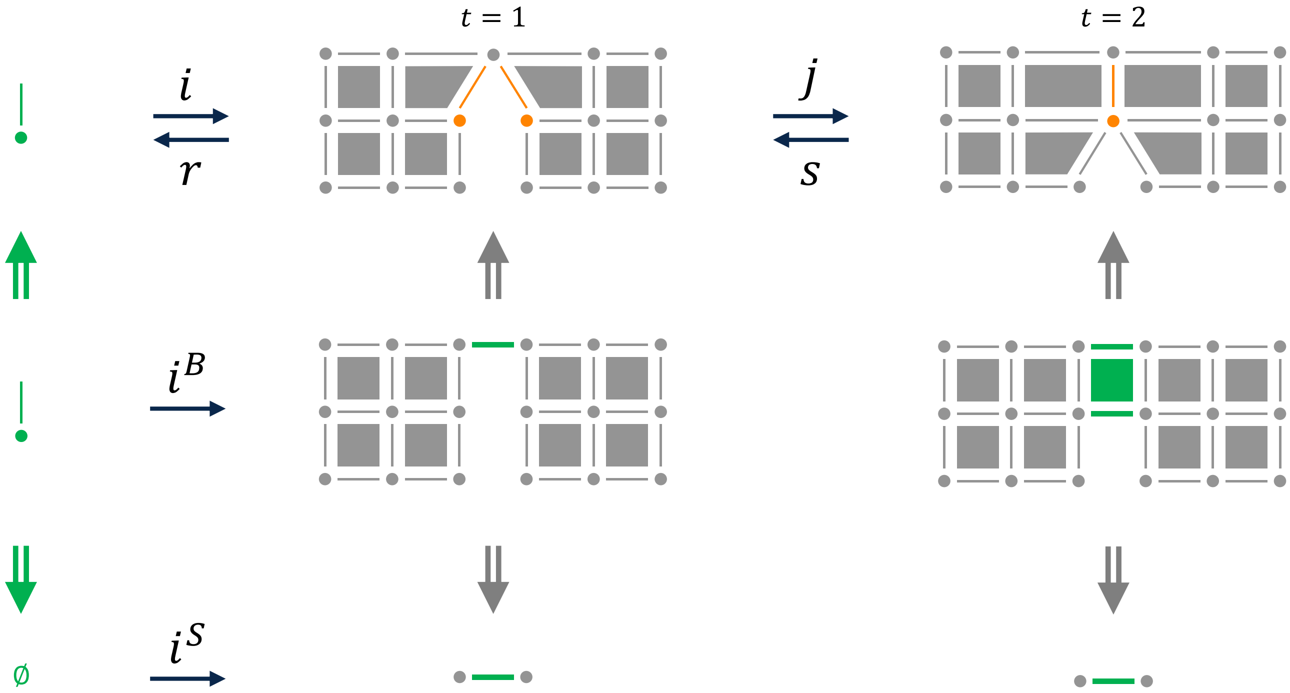

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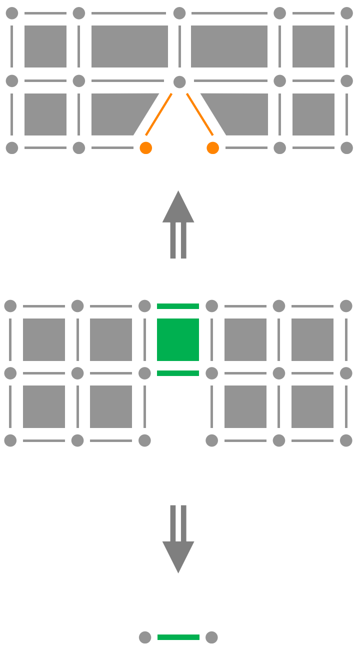
METHOD: WHEN MERGING CELLS

$t = 2$



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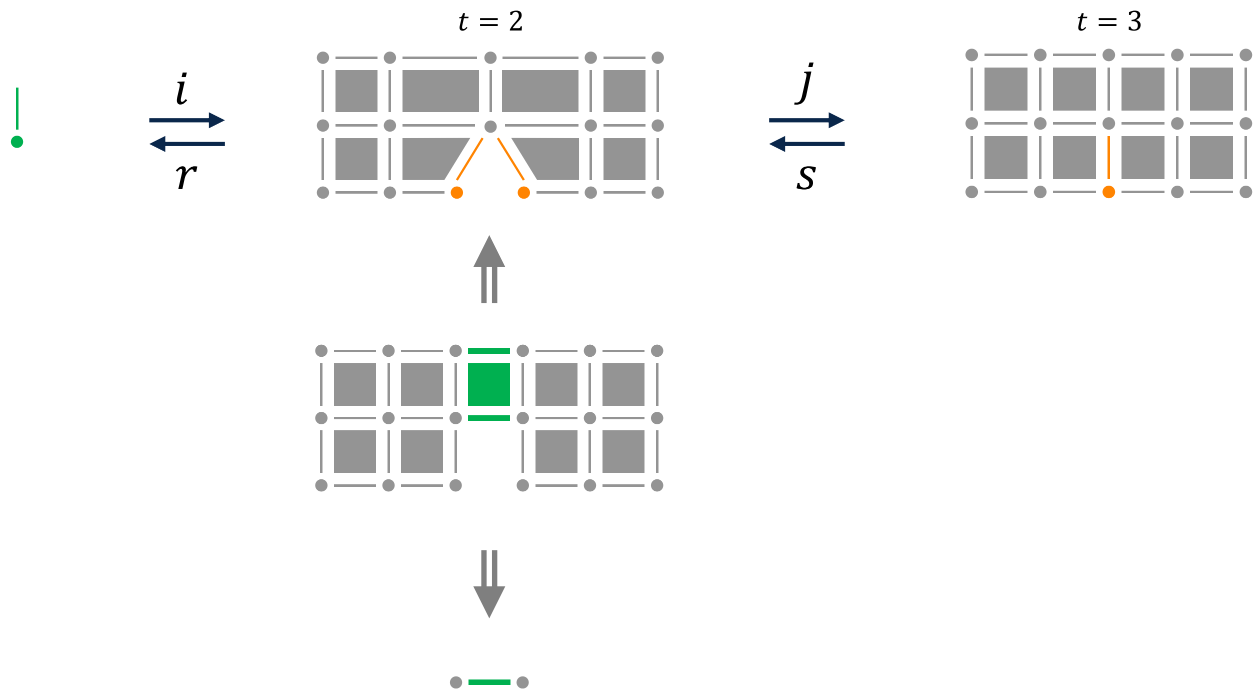

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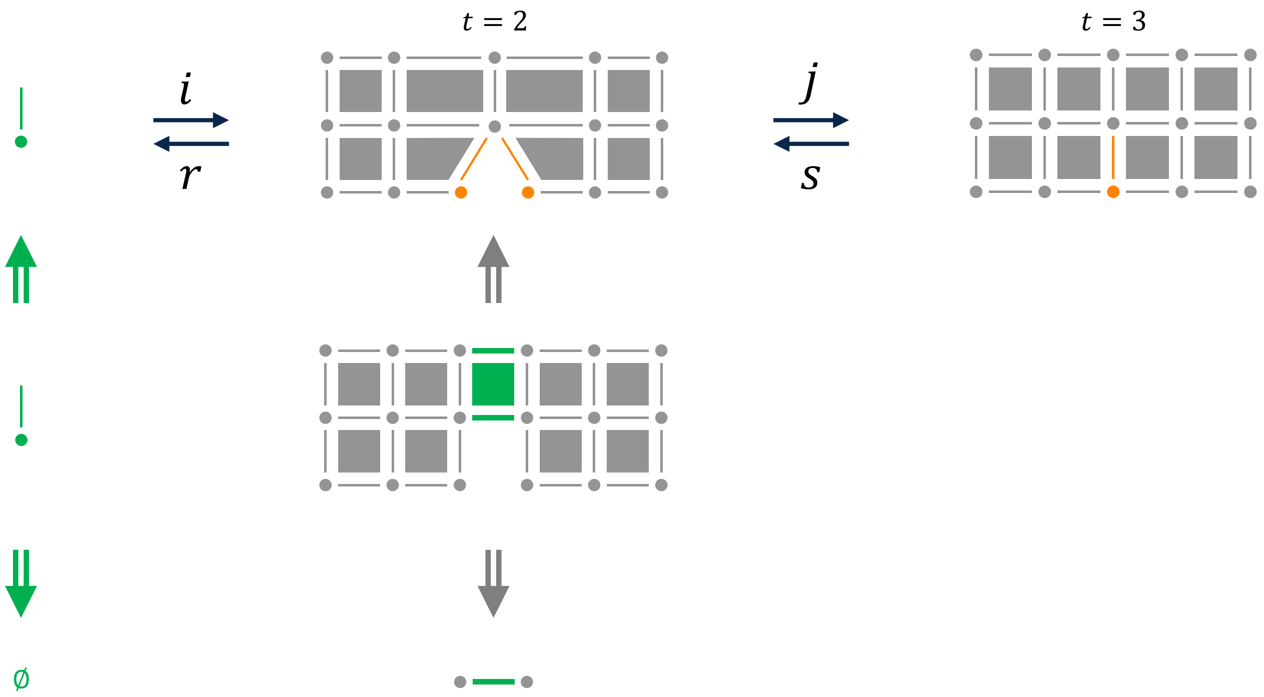

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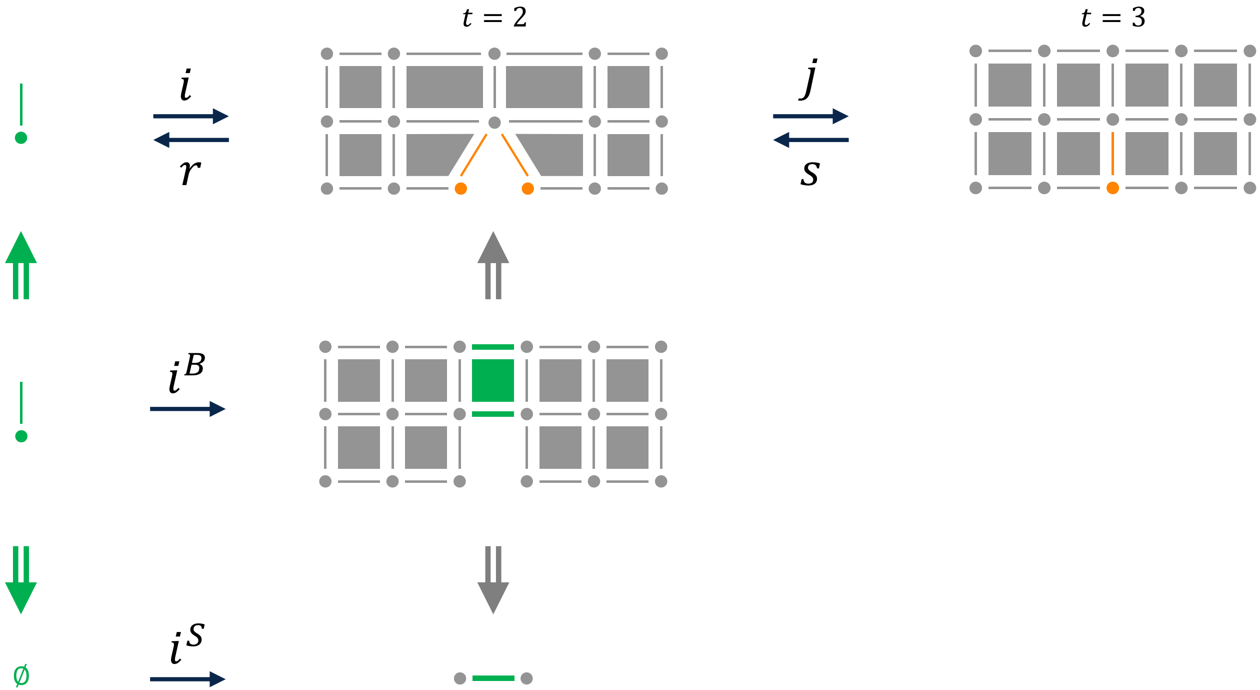
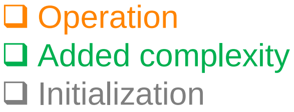
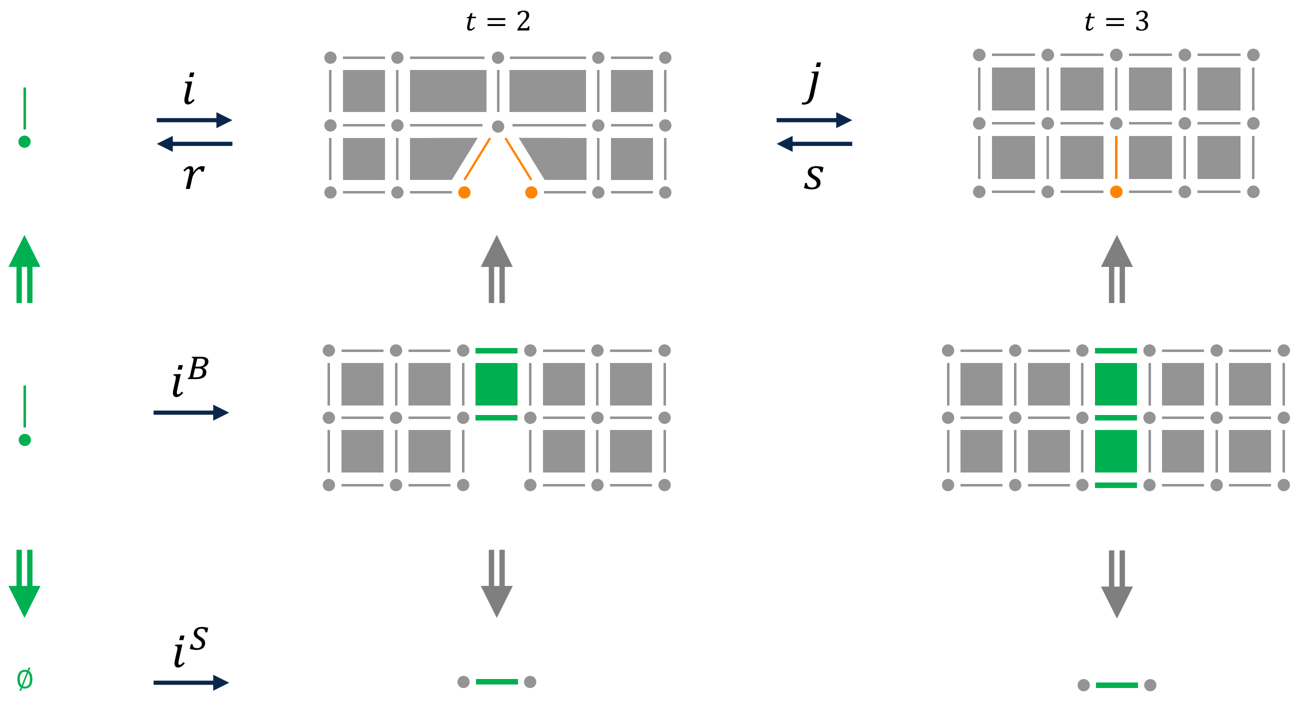




4

METHOD: WHEN MERGING CELLS



❑ Operation
❑ Added complexity
❑ Initialization

5: ANALYSIS

5

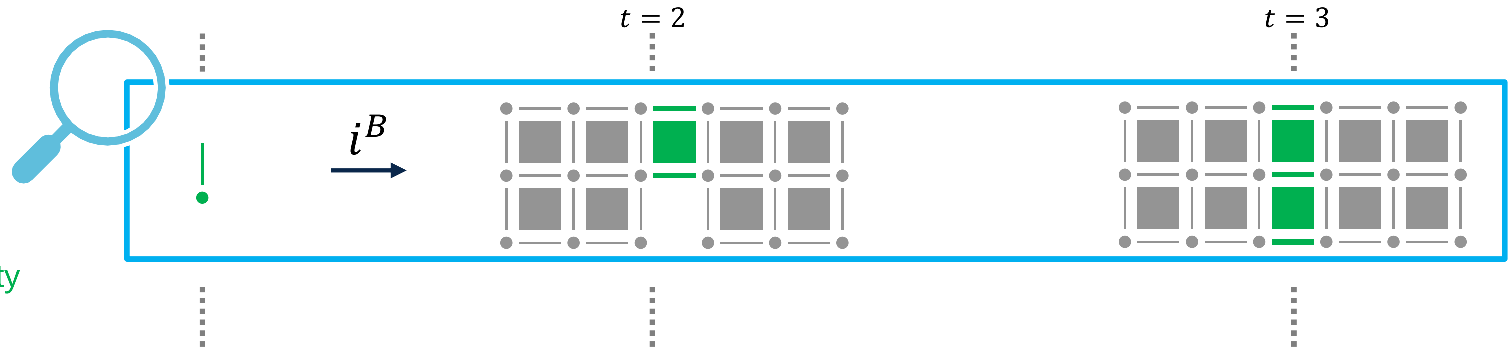
ANALYSIS

Is the computation complexity only related to the complexity of the operated part?

5

FOCUS ON INTERMEDIATE OBJECTS

At step t :



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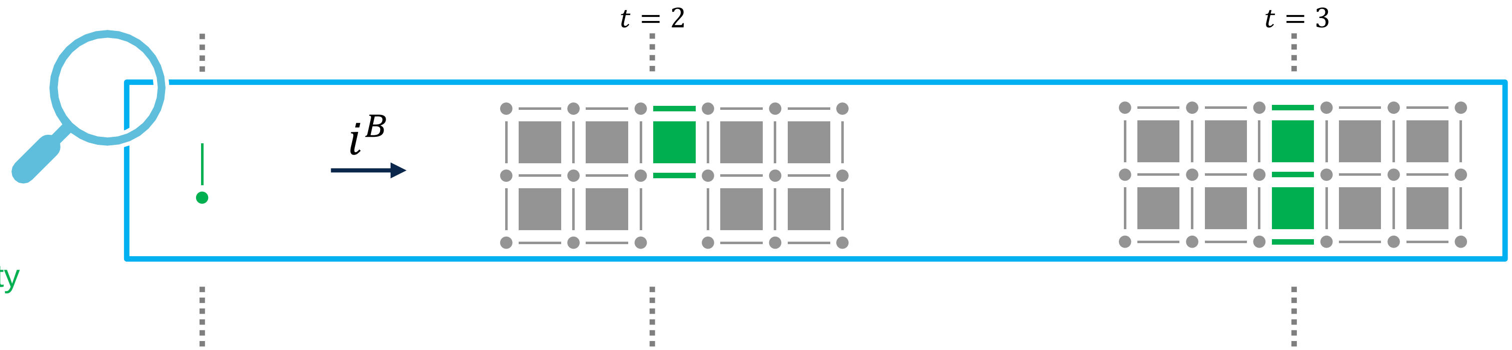
FOCUS ON INTERMEDIATE OBJECTS

At step t :

- Space: object at step t = initial object + $\sum_{n=1}^{t}$ operations representations



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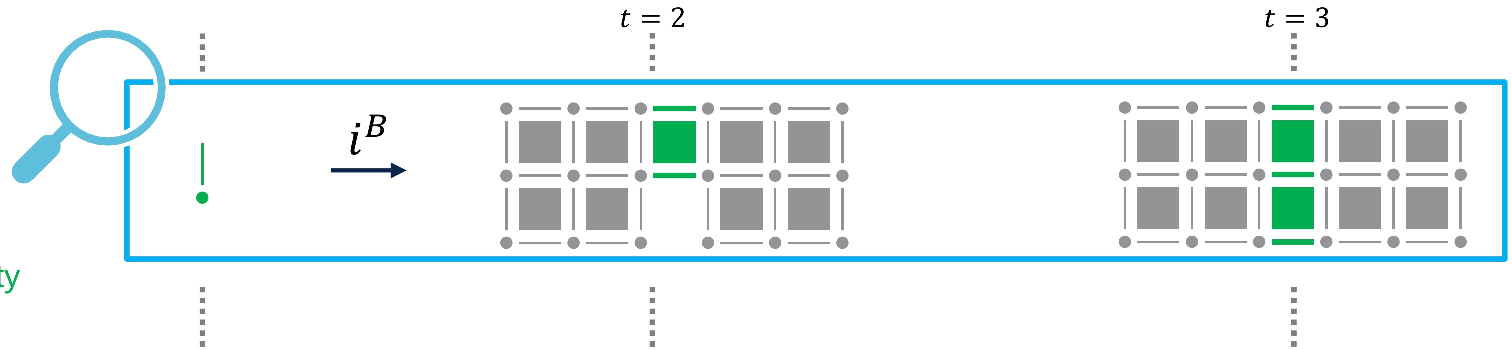

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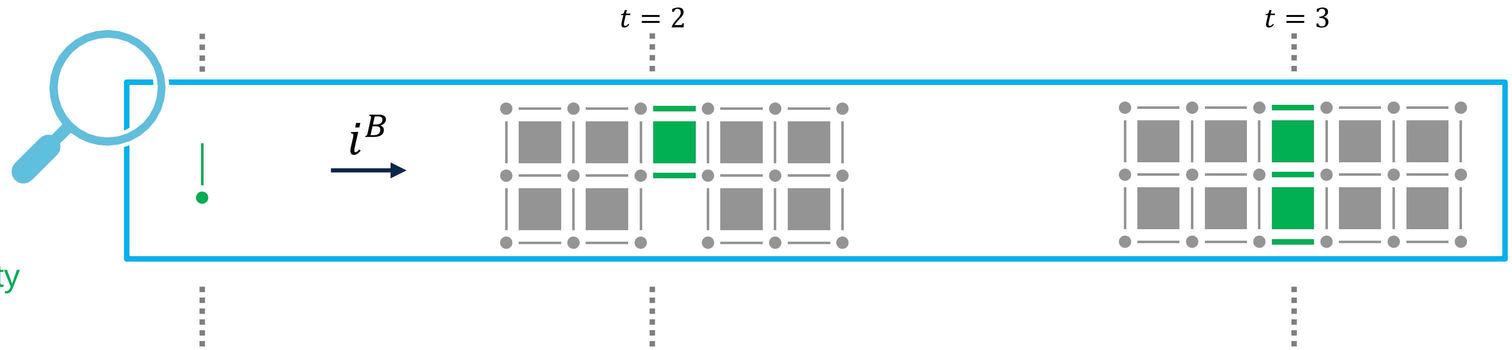

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❑ Initialization

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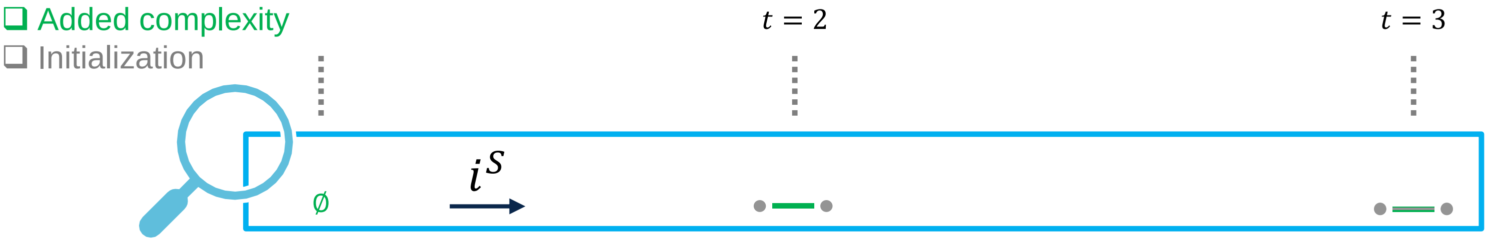
FOCUS ON REDUCED OBJECTS

EXACTLY THE SAME BEHAVIOR than intermediate objects.



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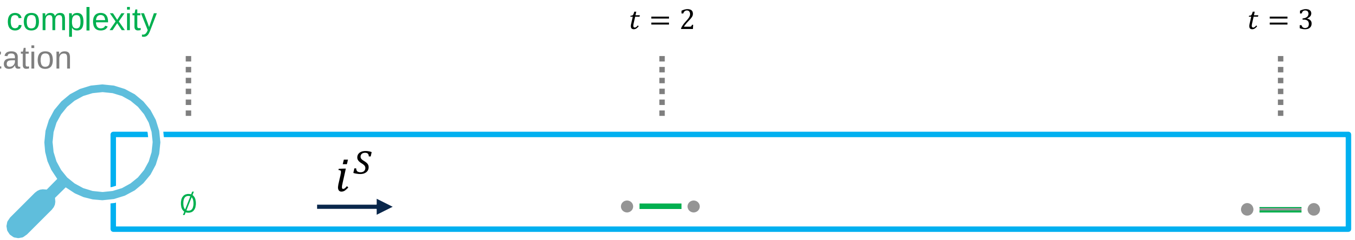
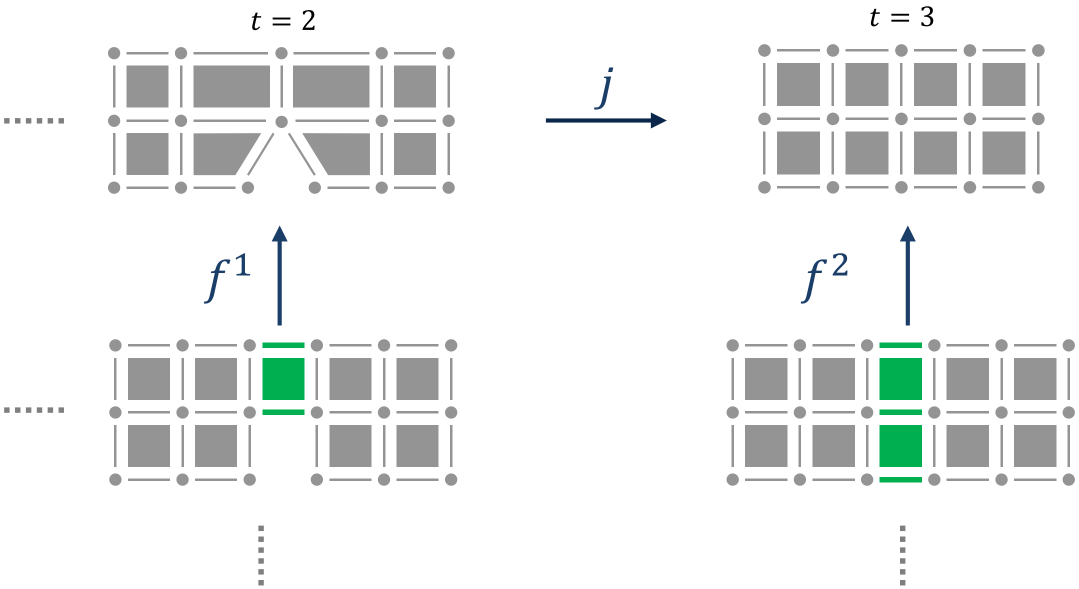
❑ Added complexity
❑ Initialization



5 FOCUS ON f COMPLEXITY

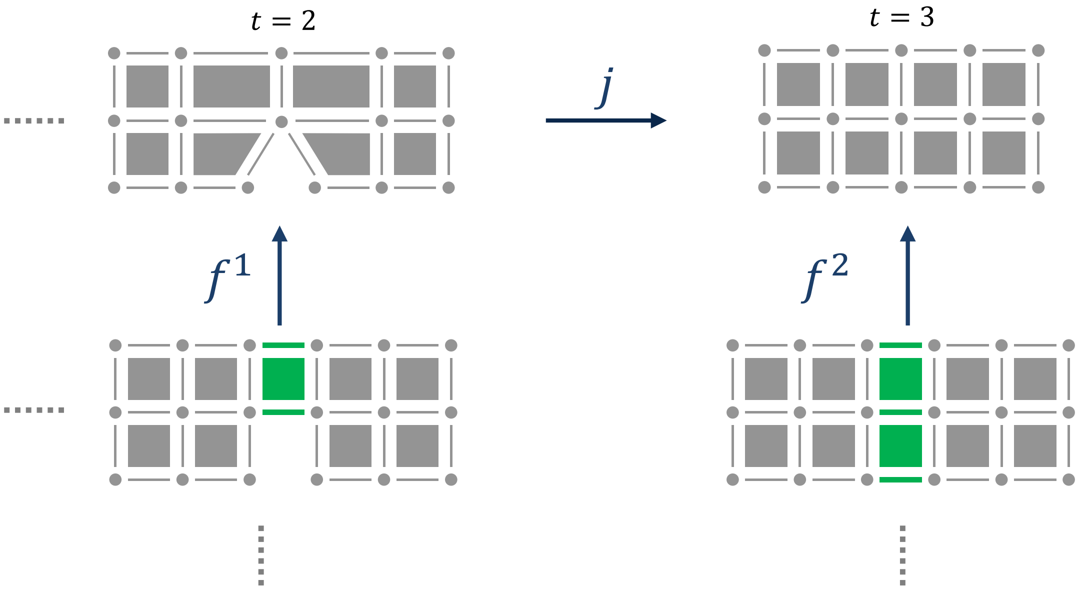


- Added complexity
- Initialization complexity

5

FOCUS ON f COMPLEXITY

$$f^2 = \binom{0}{f^1 j}$$



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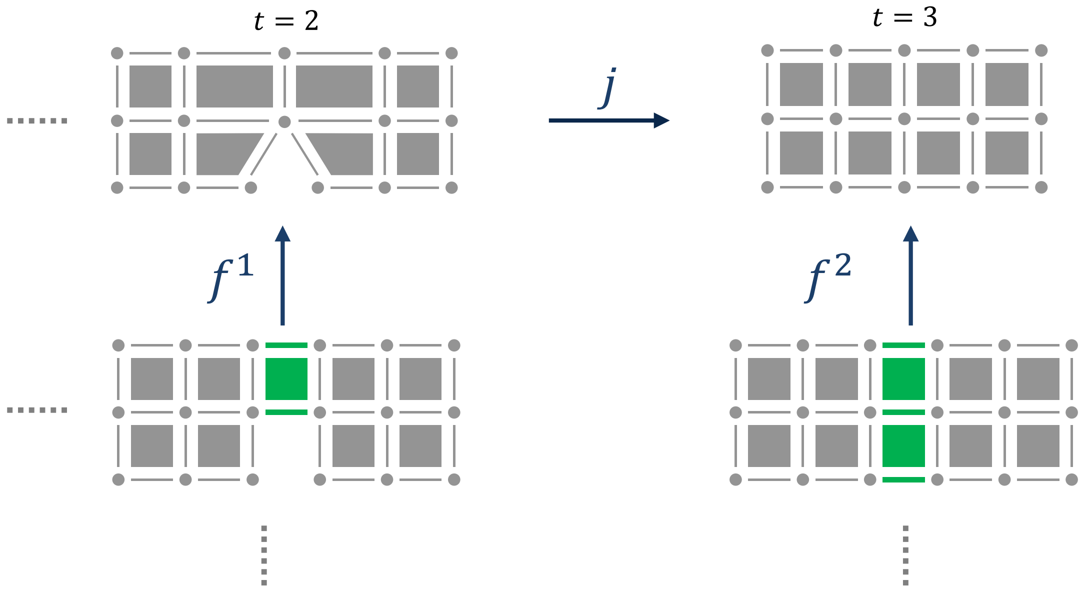

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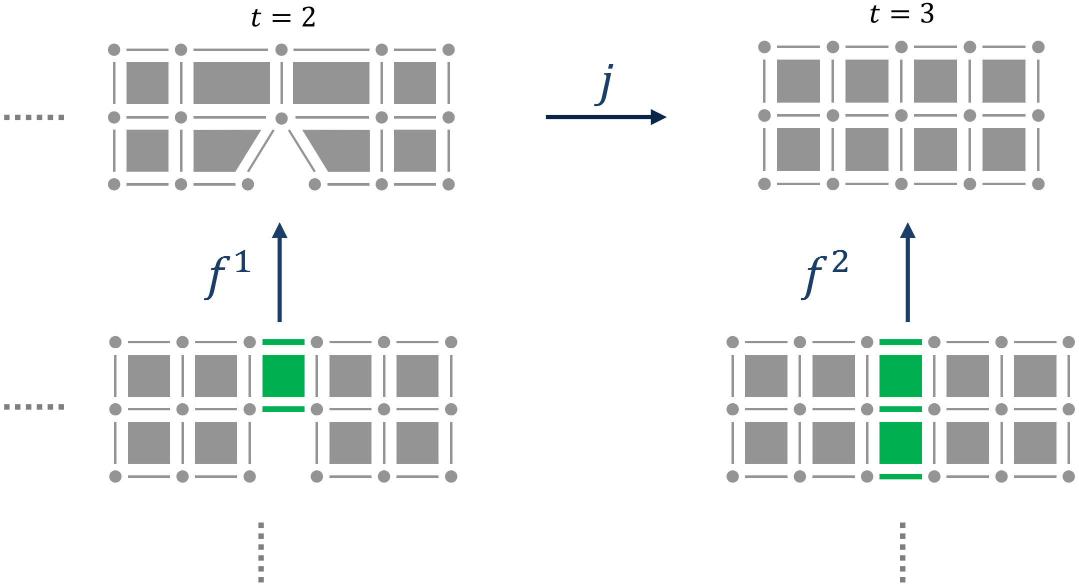

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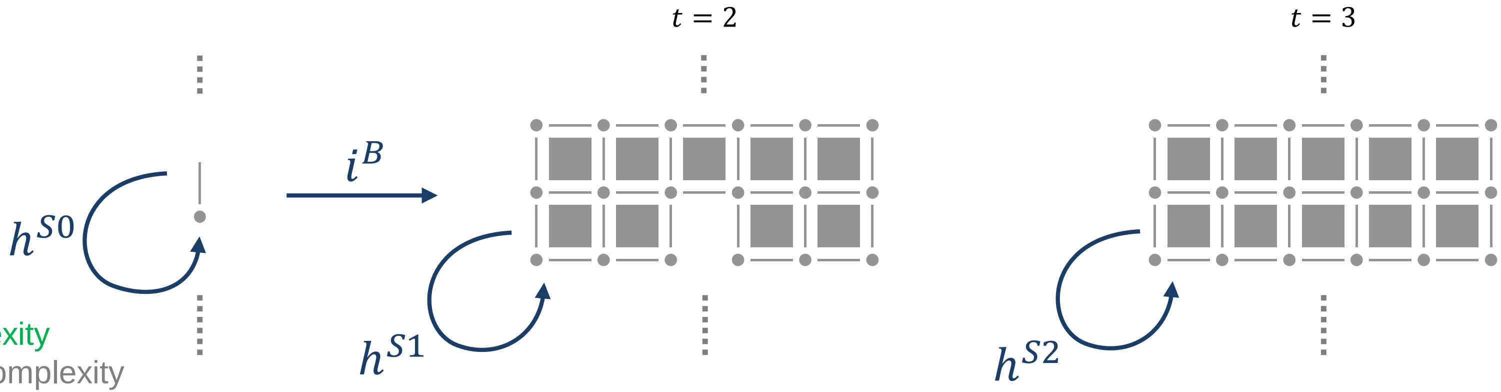
FOCUS ON h^S COMPLEXITY



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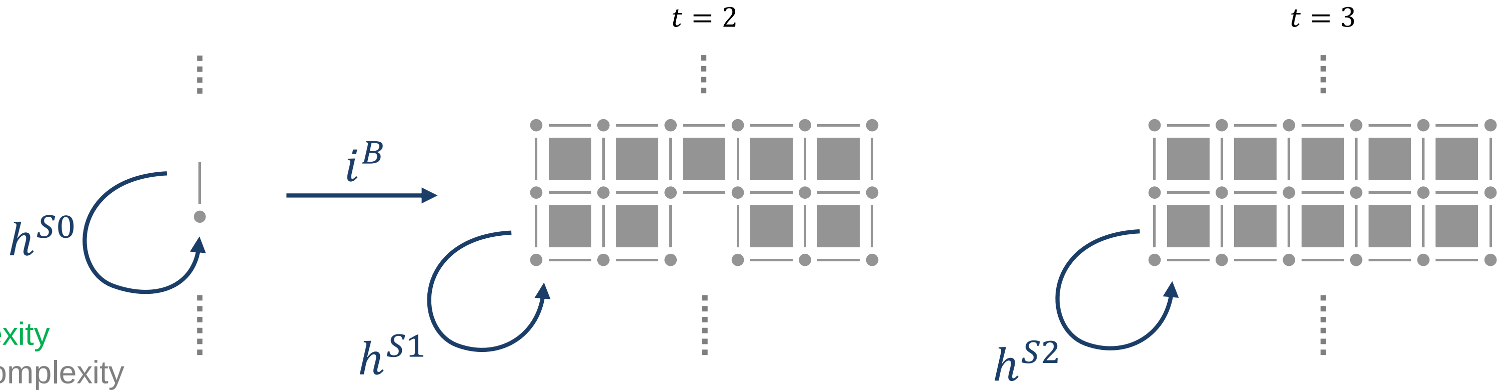
$$h^{S2} = \begin{pmatrix} -h^{S0} & h^{S0} i^B h^{S1} \\ 0 & h^{S1} \end{pmatrix}$$



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Possible loss of locality: browsing h^{S1} for h^{S2} construction

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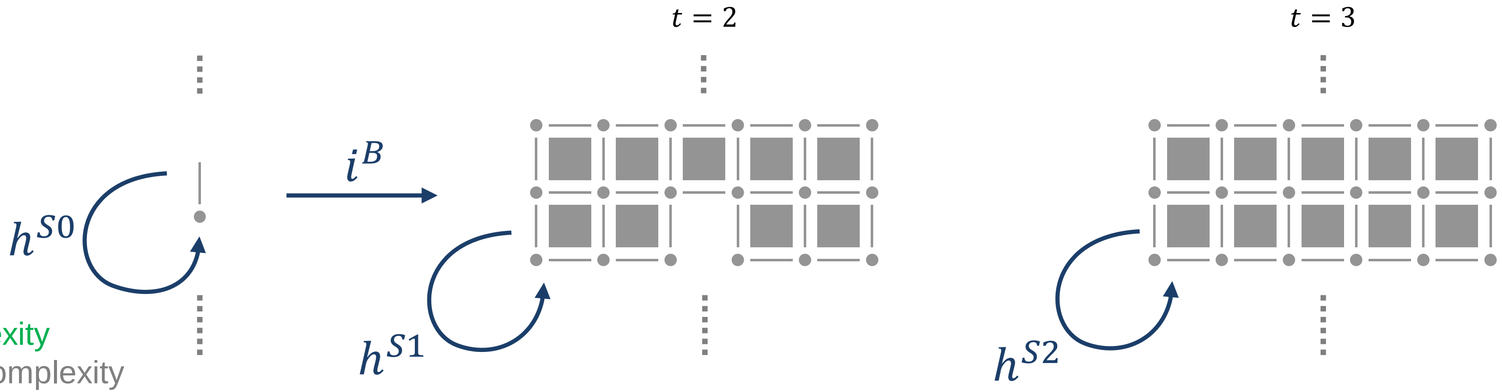

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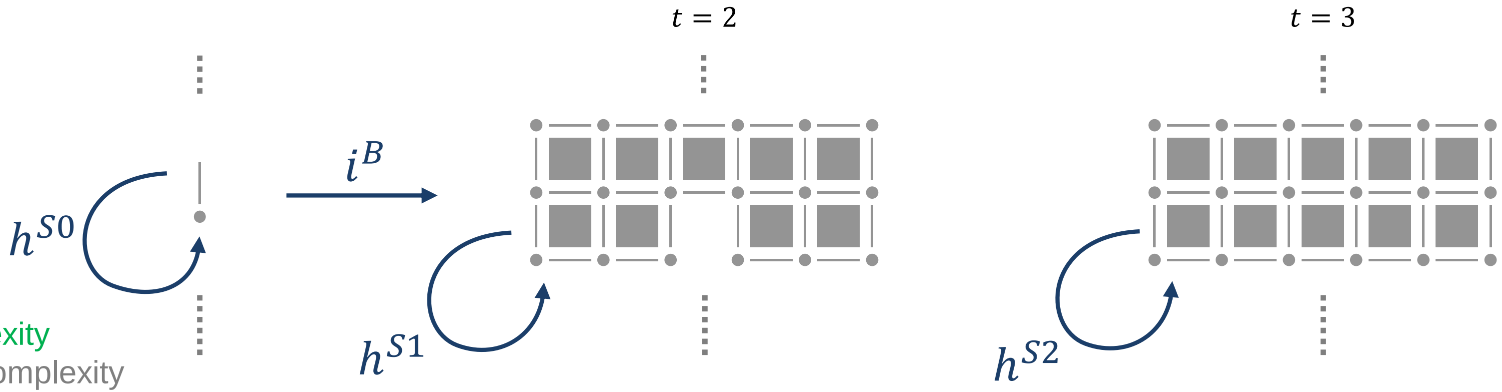

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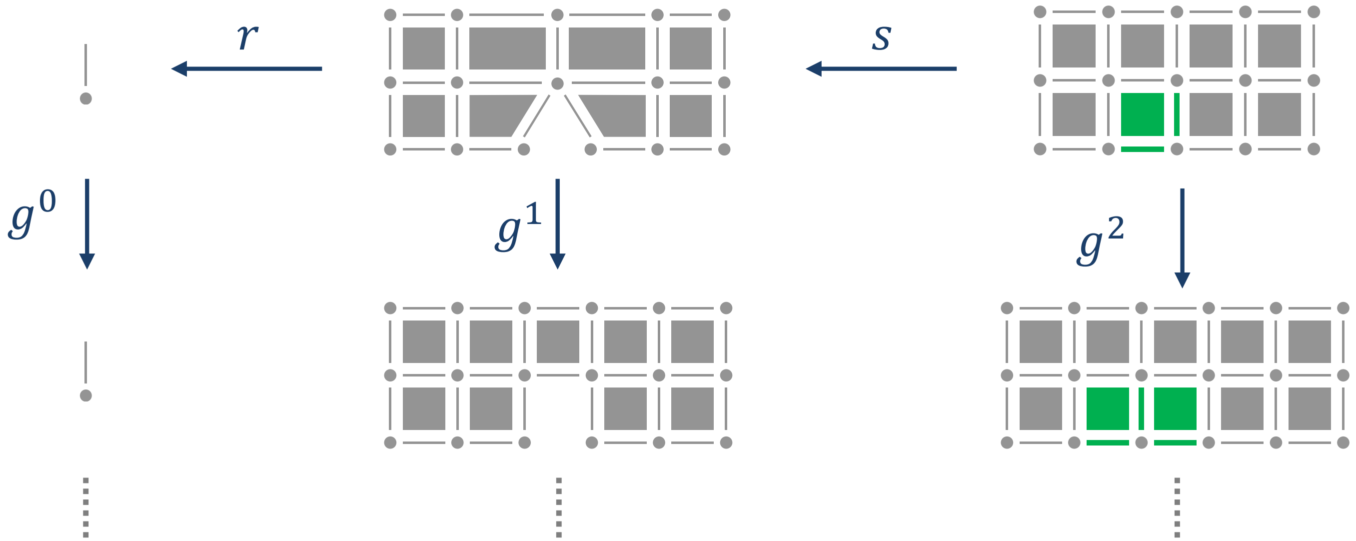

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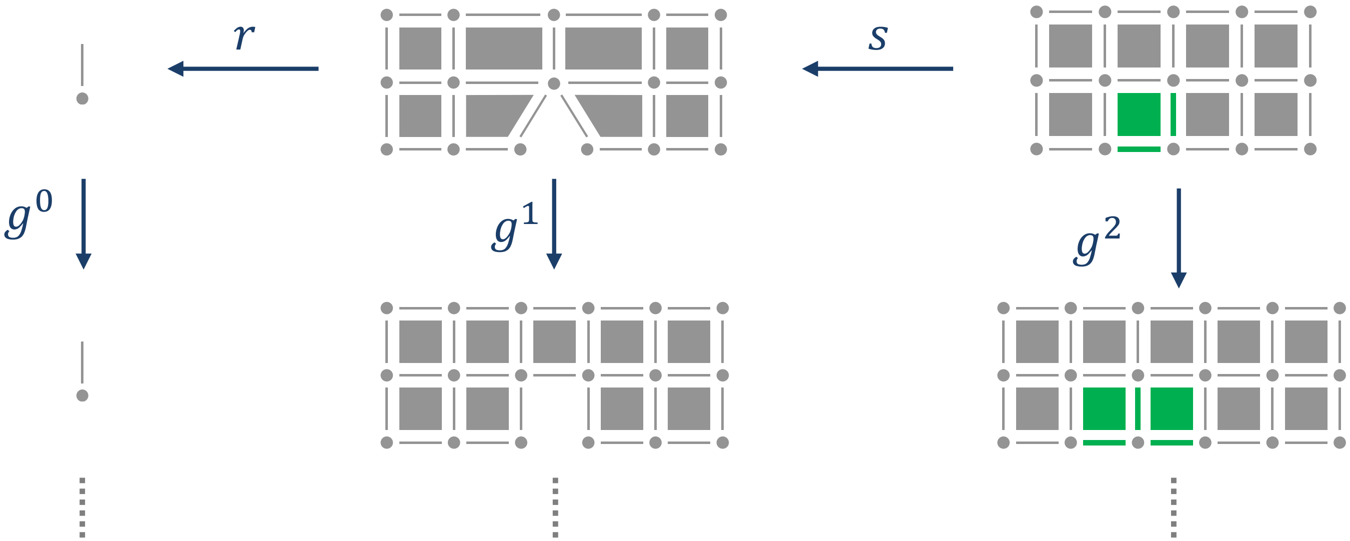
FOCUS ON g COMPLEXITY

$$g^2 = (-s\partial^1 r g^0 \quad s g^1)$$



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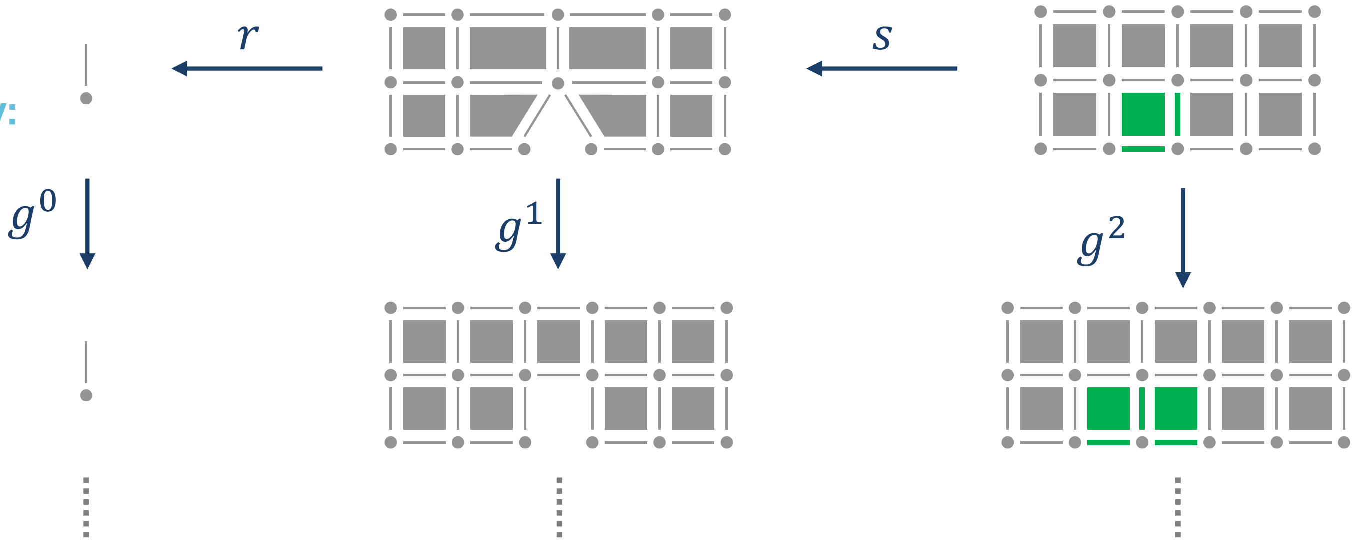

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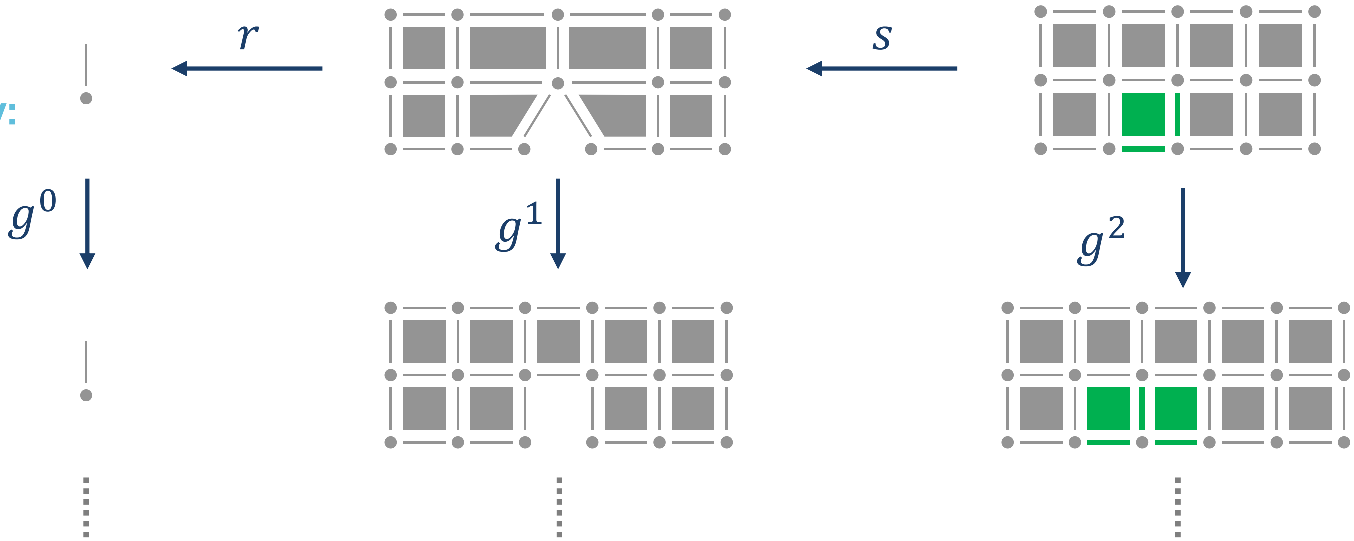

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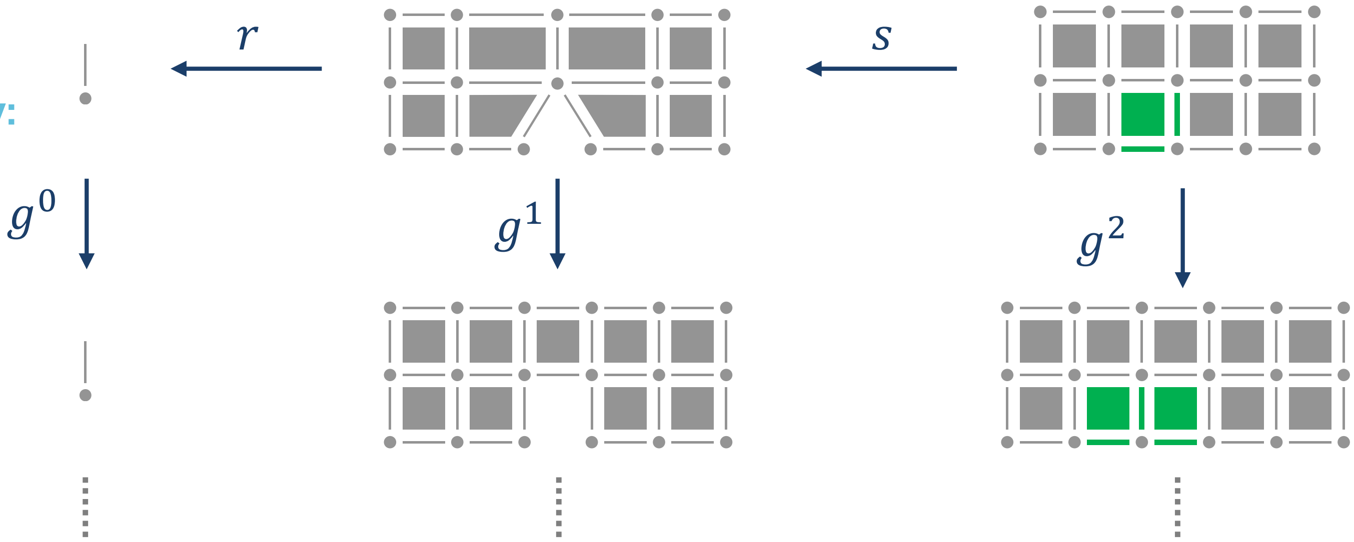

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- Handling sparsity

5

TRIED SOLUTIONS

SOFTWARE	IMPLICIT IDENTITY
EIGEN	
INTELMKL	
NUMPY	
BOOST	

5 TRIED SOLUTIONS

SOFTWARE	IMPLICIT IDENTITY	$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ BROWSING ONLY A, B, C
EIGEN		
INTELMKL		
NUMPY		
BOOST		

Essentially due to sparsity representation schemes

5 TRIED SOLUTIONS

Software	Implicit identity	$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ browsing only A, B, C	Sparse matrices
Eigen			
IntelMKL			
Numpy			
Boost			

5

PROPOSED SOLUTION

Doubly linked matrices + (i, j, v)

$$\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 4 & 0 & 3 & 4 \end{pmatrix}$$

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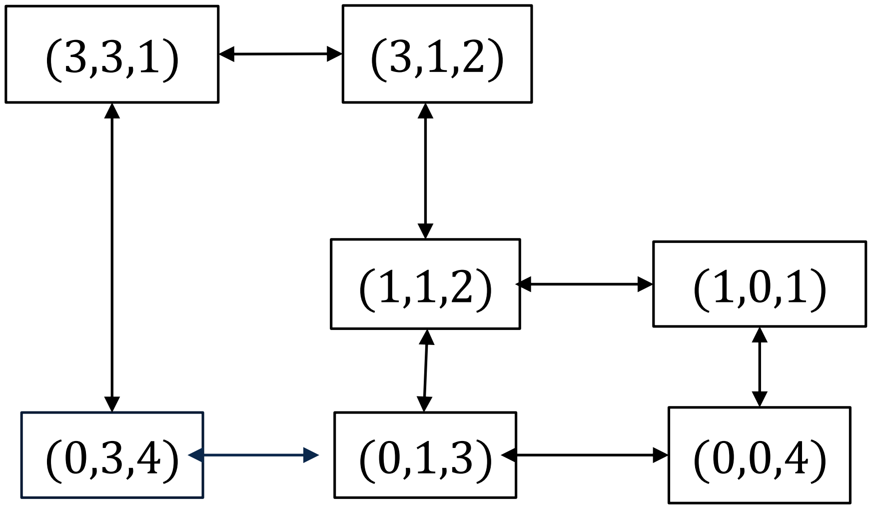
i (↑), j (←)



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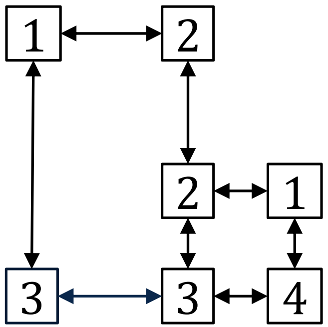

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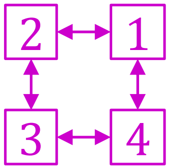

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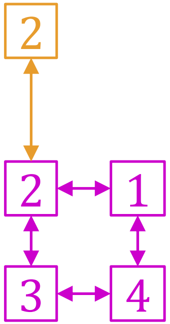

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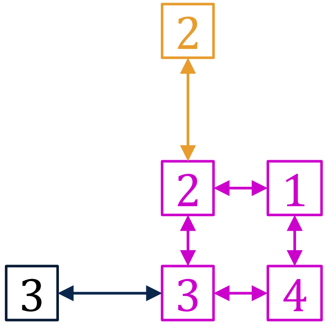

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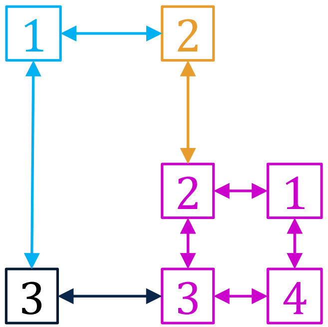

5

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Doubly linked matrices : construct $\begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{pmatrix}$ with an algorithm browsing A, B, C only

Implicit representation of identity

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow (0,0,3)$$

5

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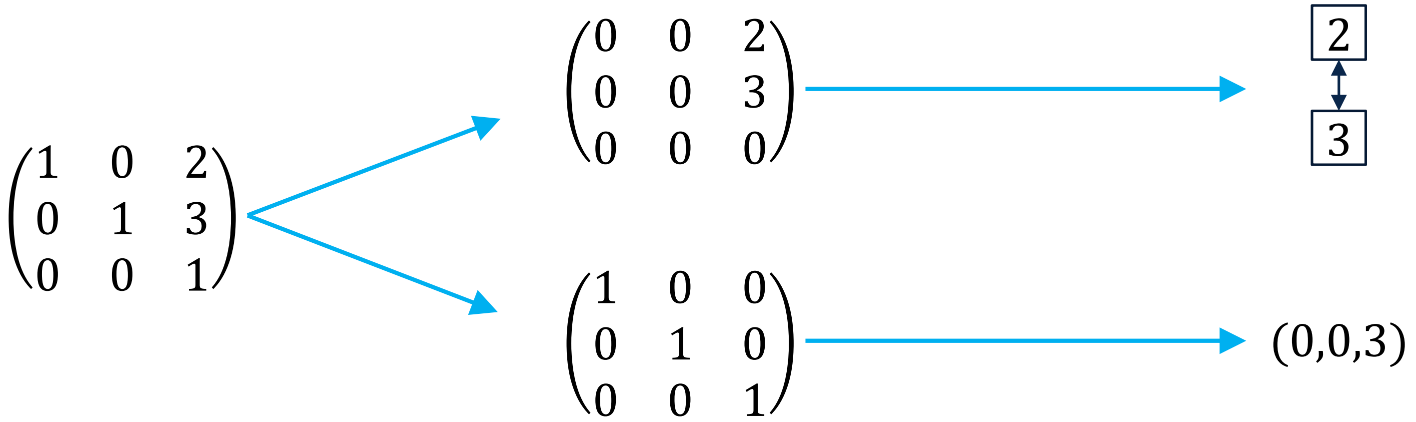

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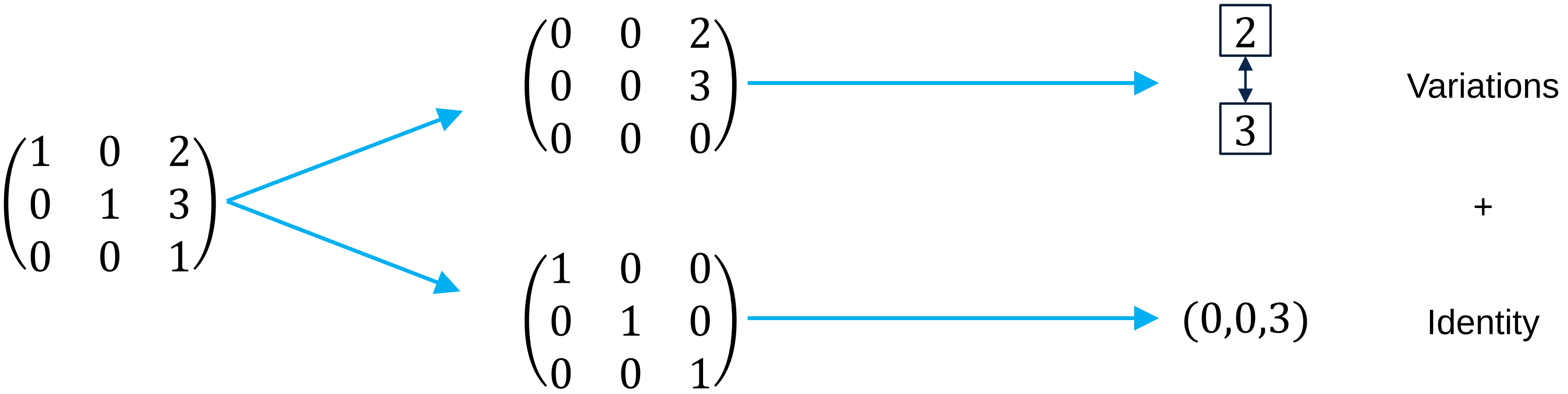
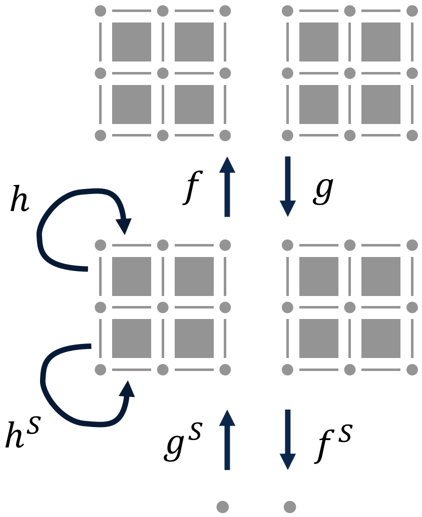


6: CONCLUSION AND PERSPECTIVES

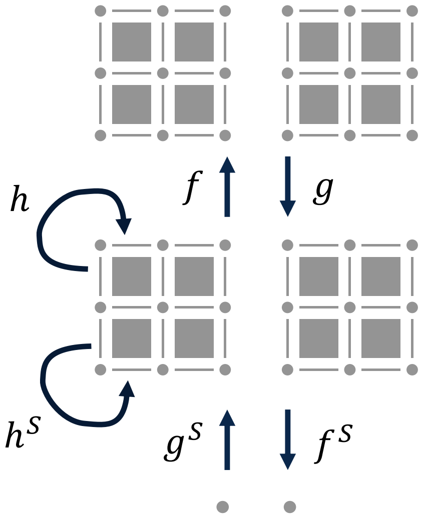
6

MORE EFFICIENT WITHOUT GENERATORS



MORE EFFICIENT WITHOUT GENERATORS

Some morphisms can be ignored



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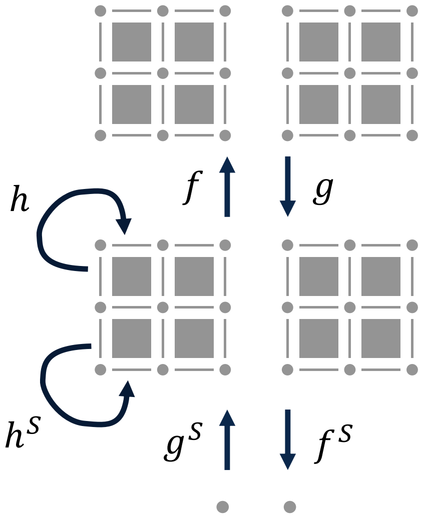

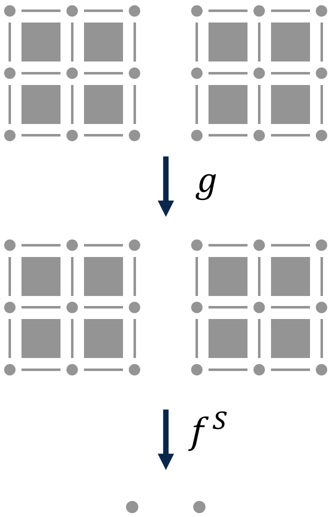

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At each step: control of the object construction according to computed information

- Short exact sequence based method => tracking homology variations

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Perspectives

- Apply the method to the inverse operation (splitting cells)

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- Apply the method to the inverse operation (splitting cells)
- Apply in an application case: animation
- Parallelization

GARBAGE

7

ATTACHMENTS

Sparse matrices representation schemes

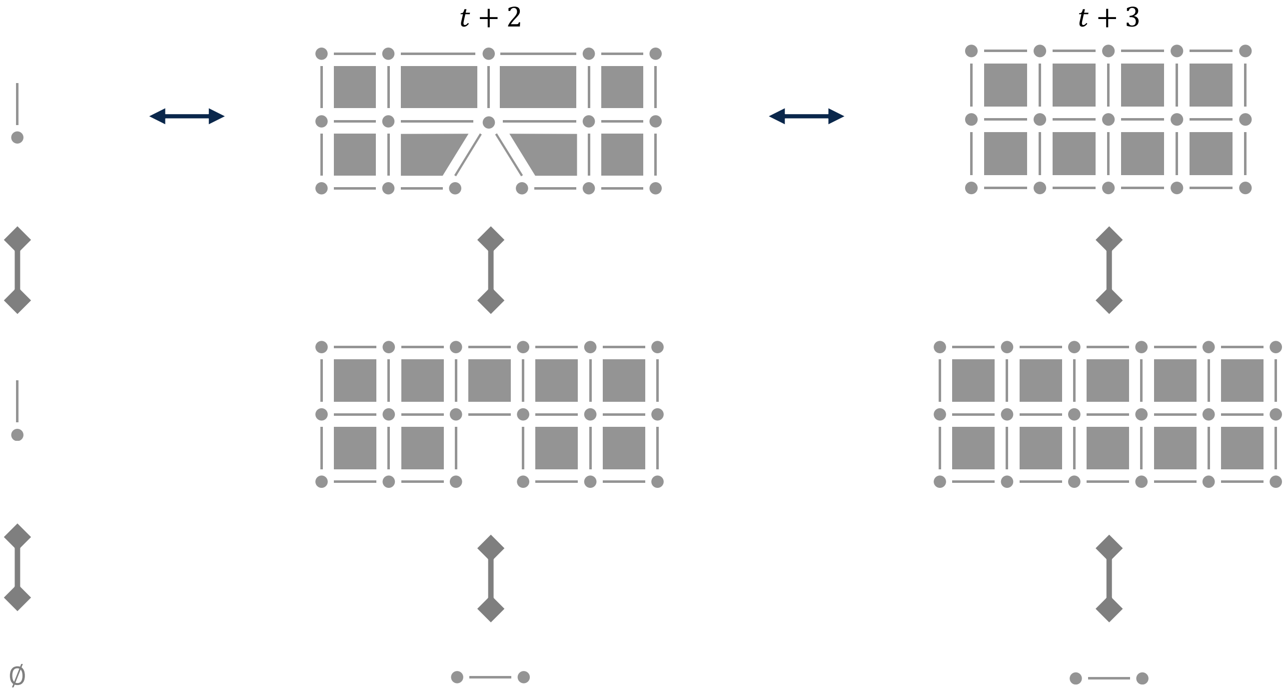
$$\begin{pmatrix} 0 & 4 & 1 \\ 2 & 0 & 5 \\ 0 & 3 & 0 \end{pmatrix}$$

CCS :

Values	2	4	3	1	5
Row Index	1	0	2	0	1
Column start index	0	1	3	6	

5

ANALYSIS : SPATIAL COMPLEXITY EVOLUTION ON COMPLEXES



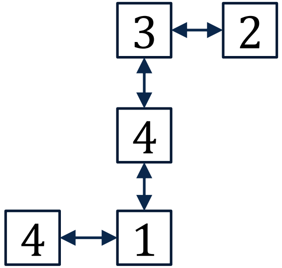
- Operation
- Added part
- Initialization

5 PROPOSED SOLUTION

Implicit identity

$$\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 0 \\ 4 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 2 \\ 0 & 4 & 0 \\ 4 & 1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow (0,0,2)$$

- ❑ Identity block