Table of contents

A combinatorial "Veronese-type" lifting

Xavier Goaoc * [1]

[1] Laboratoire Lorrain de Recherche en Informatique et ses Applications – Institut National de Recherche en Informatique et en Automatique, Université de Lorraine, Centre National de la Recherche Scientifique : UMR7503 – France

Several geometric questions can be reduced to "Here is a semi-algebraic set, is it empty?" simply by describing the set of candidate configurations that avoid the geometric property under consideration. One useful trick when working with semi-algebraic sets is the Veronese map, which "lifts" a question about degree-k polynomials in Rd to a question about linear forms (in the space of monomials of degree =< k in d variables); a beautiful illustration is the polynomial ham-sandwich. While working on a problem in discrete geometry (geometric permutations), we came across a semi-algebraic system that can be efficiently solved via a combinatorial analogue of the Veronese map. This is joint work with Andreas Holmsen (KAIST) and Cyril Nicaud (LIGM).

*Speaker

Approximating k-fold filtrations using weighted Delaunay triangulations

Mickaël Buchet * [1], Michael Kerber [1]

[1] TU Graz, Institute for Geometry – Austria

k-fold filtrations are one option to handle noise when computing persistent homology. These filtrations are obtained as the k-cover of a union of balls when the radius of these balls increases. We study some ways to approximate the k-fold filtration using a union of weighted balls. The topology of this union can be studied using the associated weighted Delaunay triangulation. We provide two different methods to do such approximation and look into the properties of the resulting power diagrams obtained as the dual of the weighted Delaunay triangulations.

Keywords: k, cover, approximation, weighted Delaunay triangulations, power diagrams

*Speaker

Scaffolding skeletons using spherical Voronoi diagrams: feasibility, regularity and symmetry.

Alvaro Fuentes * [1], Evelyne Hubert [2]

[1] AROMATH team, Inria Sophia Antipolis – L'Institut National de Recherche en Informatique et e n Automatique (INRIA) – France
[2] AROMATH team, Inria Sophia Antipolis – L'Institut National de Recherche en Informatique et e n Automatique (INRIA) – 2004 route des Lucioles, 06902 Sophia Antipolis, France., France

Given a skeleton made of line segments we describe how to obtain a coarse quad mesh of a surface that encloses tightly the skeleton and follows its structure - the *scaffold* . We formalize as an Integer Linear Program the problem of constructing an optimal scaffold that minimizes the total number of quads on the mesh. We prove the feasibility of the Integer Linear Program for any skeleton. In particular we can generate these scaffold for skeletons with cycles. We additionally show how to obtain *regular* scaffold, i.e. with the same number of quad patches around each line segment, and *symmetric* scaffold that respect the symmetries of the skeleton.

*Speaker

Approximate strong edge-colouring of unit disk graphs

Nicolas Grelier * [1], Rémi De Joannis De Verclos [2], Ross Kang [2], François Pirot [3,2]

[1] Department of Computer Science, ETH Zurich – Switzerland
[2] Department of Mathematics, Radboud University – Netherlands
[3] LORIA – Institut National de Recherche en Informatique et en Automatique, Université de Lorraine – France

We show that the strong chromatic index of unit disk graphs is efficiently 6-approximable. This improves on 8-approximability as shown by Barrett, Istrate, Kumar, Marathe, Thite, and Thulasidasan (2006). We also show that strong edge-6-colourability is NP-complete for the class of unit disk graphs. Thus there is no polynomial-time $(7/6-\epsilon)$-approximation unless P=NP.

Keywords: unit disk graphs, graph colouring, approximation algorithms

*Speaker

Local computation of homology variations related to cell merging: theoretical results and implementation issues

Wassim Rharbaoui * [1], Sylvie Alayrangues [1], Samuel Peltier [1], Pascal Lienhardt [1]

[1] XLIM – Centre National de la Recherche Scientifique : UMR7252, Université de Poitiers – 123 Avenue Albert THOMAS 87060 LIMOGES CEDEX, France

In this talk, we study the complexity of a homology computation method designed to track homology variations over a construction process composed of a succession of cells merging operation. We show, through theoretical and experimental results, that this complexity is "local", meaning that it depends on "the operation
size" rather than the object size. We also expose cautions that have to be taken regarding the implementation to preserve this locality.

Keywords: Homology, algebraic topology

*Speaker

Hard problems in knot theory

Arnaud De Mesmay * [1], Yo'av Rieck [2], Eric Sedgwick [3], Martin Tancer [4]

[1] CNRS, GIPSA-lab – CNRS : UMR5216, Gipsa-lab – France
[2] University of Arkansas [Fayetteville] – United States
[3] DePaul University – United States
[4] Charles University [Prague] – Czech Republic

Quite a few problems in knot theory are extremely hard to solve algorithmically (like testing whether two knots are equivalent), and some of them are not even known to be decidable (like computing the unknotting number of a knot). However, very few hardness results are known. We show how a rather simple construction with Borromean rings can be leveraged to establish a handful of NP-hardness proofs for seemingly unrelated problems. Our main result shows that deciding if a diagram of the unknot can be untangled using at most k Riedemeister moves (where k is part of the input) is NP-hard. We also prove that several natural questions regarding links in the 3-sphere are NP-hard, including detecting whether a link contains a trivial sublink with n components, computing the unlinking number of a link, and computing a variety of link invariants related to four-dimensional topology (such as the 4-ball Euler characteristic, the slicing number, and the 4-dimensional clasp number).

Keywords: knot theory, Reidemeister moves, hardness, unknot recognition

*Speaker

Local conditions for triangulating submanifolds of Euclidean space

Andre Lieutier * [1], Jean-Daniel Boissonnat [2], Mathijs Wintraecken [2], Ramsay Dyer , Ghosh Arijit

[1] Dassault Systèmes – Dassault Systèmes – France
[2] Université Cote d'Azur, Institut National de Recherche en Informatique et en Automatique, Centre Sophia-Antipolis Méditerranée (INRIA SAM) – Université Cote d'Azur, Institut National de Recherche en Informatique et en Automatique, Centre Sophia-Antipolis Méditerranée – 2004 Route des Lucioles 06902 Valbonne, France

One says that a simplicial complex triangulates a manifold when it is homeomorphic to it. We consider the situation where M is an m-dimensional submanifold of Euclidean space with positive reach

and A a pure m-dimensional simplicial complex whose vertices are points in M.

In this context, several proofs of submanifold triangulation start by establishing local properties.

Typically:

* simplices are not too "flat" and are small with respect to the submanifold reach,

* two m-dimensional nearby simplices have interior disjoint projections on local tangent planes,

* the simplicial complex A has no boundary in the sense that any simplex has at least two m-dimensional cofaces.
We show that, one quantified, these conditions are sufficient to state in fact an ambient isotopy between A and M.

Keywords: manifold, triangulation, topology, isotopy

*Speaker

Triangulating submanifolds: An elementary and quantified version of Whitney's method

Jean-Daniel Boissonnat [1], Siargey Kachanovich , Mathijs Wintraecken * [1]

[1] Université Cote d'Azur, Institut National de Recherche en Informatique et en Automatique, Centre Sophia-Antipolis Méditerranée (INRIA SAM) – Université Cote d'Azur, Institut National de Recherche en Informatique et en Automatique, Centre Sophia-Antipolis Méditerranée – 2004 Route des Lucioles 06902 Valbonne, France

The first insightful geometric and constructive proof of the existence of a triangulation of any smooth manifold was given by Hassler Whitney. We quantize Whitney's construction (in terms of the reach of the manifold) to prove the existence of a triangulation for any C^2 manifold, so that we get an algorithm with explicit bounds. We also give a new elementary proof, which is completely geometric.

Keywords: Triangulation, reach, manifold, Whitney

*Speaker

Expected Complexity of Routing in Theta-6 and Half-Theta-6 Graphs

Olivier Devillers * [1]

[1] Laboratoire Lorrain de Recherche en Informatique et ses Applications – Institut National de Recherche en Informatique et en Automatique, Université de Lorraine, Centre National de la Recherche Scientifique : UMR7503 – France

We provide an analysis of the average routing ratio of routing algorithms in Theta_6 and Half-Theta_6 in Poisson point process. The basic Theta routing has an average ratio of 1.165 while the positive routing has an average ratio of 1.078. Half-Theta_6 has only half of the edges of Theta_6, for this graph we propose two new routing algorithms with the same worst-case ratio than the previously known algorithm and whose expected ratio are 1.569 and 1.265. The first of this routing is memoryless and the second one needs to remember the origin of the message. All these expected ratio can be expressed in term of the angle between the line through the source and target of the routing and the axis that determine the Theta graph.

Keywords: TD, Delaunay, Probabilistc Analysis

*Speaker

Numerical Algorithm for the Topology of Singular Plane Curves

George Krait * [1], Sylvain Lazard [1], Guillaume Moroz [1], Marc Pouget [1]

[1] Université de Lorraine, CNRS, Inria, LORIA, F-54000 Nancy, France – LORIA Nancy - CNRS, L'Institut National de Recherche en Informatique et e n Automatique (INRIA), universté de lorraine – France

We are interested in computing the topology of plane singular curves. For this, the singular points must be isolated. Numerical methods for isolating singular points are efficient but not certified in general. We are interested in developing certified numerical algorithms for isolating the singularities. In order to do so, we restrict our attention to the special case of plane curves that are projections of smooth curves in higher dimensions. In this setting, we show that the singularities can be encoded by a regular square system whose isolation can be certified by numerical methods. This type of curves appears naturally in robotics applications and scientific visualization.

Keywords: singular curves, certified numerical methods

*Speaker

Delaunay triangulations of a family of symmetric hyperbolic surfaces in practice

Matthijs Ebbens [1], Iordan Iordanov [2], Monique Teillaud * [3], Gert Vegter [1]

[1] Johann Bernoulli Institute for Mathematics and Computer Science (JBI) – Nijenborgh 9 9747 AG Groningen, Netherlands
[2] Laboratoire Lorrain de Recherche en Informatique et ses Applications (LORIA) – Centre National de la Recherche Scientifique : UMR7503, Université de Lorraine, Institut National de Recherche en Informatique et en Automatique – Campus Scientifique BP 239 54506 Vandoeuvre-lès-Nancy Cedex, France
[3] Inria Nancy - Grand Est – Institut National de Recherche en Informatique et en Automatique – 615 rue du Jardin Botanique 54600 Villers-lès-Nancy, France

The talk first presents an algorithm to compute Delaunay triangulations of the Bolza surface, the most symmetric hyperbolic surface of genus two. Then higher genus surfaces are considered. The implementation and experiments are also mentioned. A software package for the Bolza surface has been recently integrated into CGAL.

Keywords: hyperbolic surface, Delaunay triangulation

*Speaker

An implementation of the homotopy test in CGAL

Francis Lazarus * [1], Guillaume Damiand *

2

[1] CNRS – CNRS : UMR5216 – France
[2] CNRS – CNRS : UMR5207 – France

We will first recall the linear time algorithm of Erickson and Whittelsey for deciding if two curves on a combinatorial surface can be continuously deformed one into the other. We then describe our implementation in CGAL as a new package and show its simple use. We finally discuss what could be implemented in this package concerning the topology of curves on surfaces.

Keywords: CGAL, homotopy, curves on surfaces

*Speaker

Poisson sample and 3D-Delaunay on surface

Charles Duménil * [1], Olivier Devillers

[1] Université de Lorraine, CNRS, Inria, LORIA, Nancy – Université de Lorraine – France

The complexity of the 3D-Delaunay triangulation (tetrahedralization) of n points distributed on a surface ranges from linear to quadratic. When the points are a deterministic good sample

of a smooth compact generic surface, the size of the Delaunay triangulation is O(n log n).

Using this result, we prove that when points are Poisson distributed on a surface under the same
hypothesis, whose expected number of vertices is , the expected size is O(log2).

Keywords: Surface, 3D, Delaunay, Probability

*Speaker

Computing hyperbolic structures from Thurston's equations

Owen Rouille * [1], Clément Maria [1]

[1] Inria Sophia Antipolis - Méditerranée – Institut National de Recherche en Informatique et en Automatique – France

A fundamental approach for the study of knots is via the study of their geometry. More specifically, almost all knots are hyperbolic, *i.e.* when considering the complement of a knot in S^3, one gets a manifold which admits a hyperbolic geometry. Consequently, it is fundamental to represent hyperbolic structures and have efficient algorithms to find them. This talk presents a method to find a hyperbolic metric of a knot complement studying angle structures and Thurston's gluing equations. This presentation is a preliminary study essentially based on the survey "From angled triangulations to hyperbolic structures" by D. Futer and F. Guéritaud.

Keywords: Thurston's gluing equations, 3 manifold, hyperbolic metric, knot, angle structure

*Speaker

Author Index