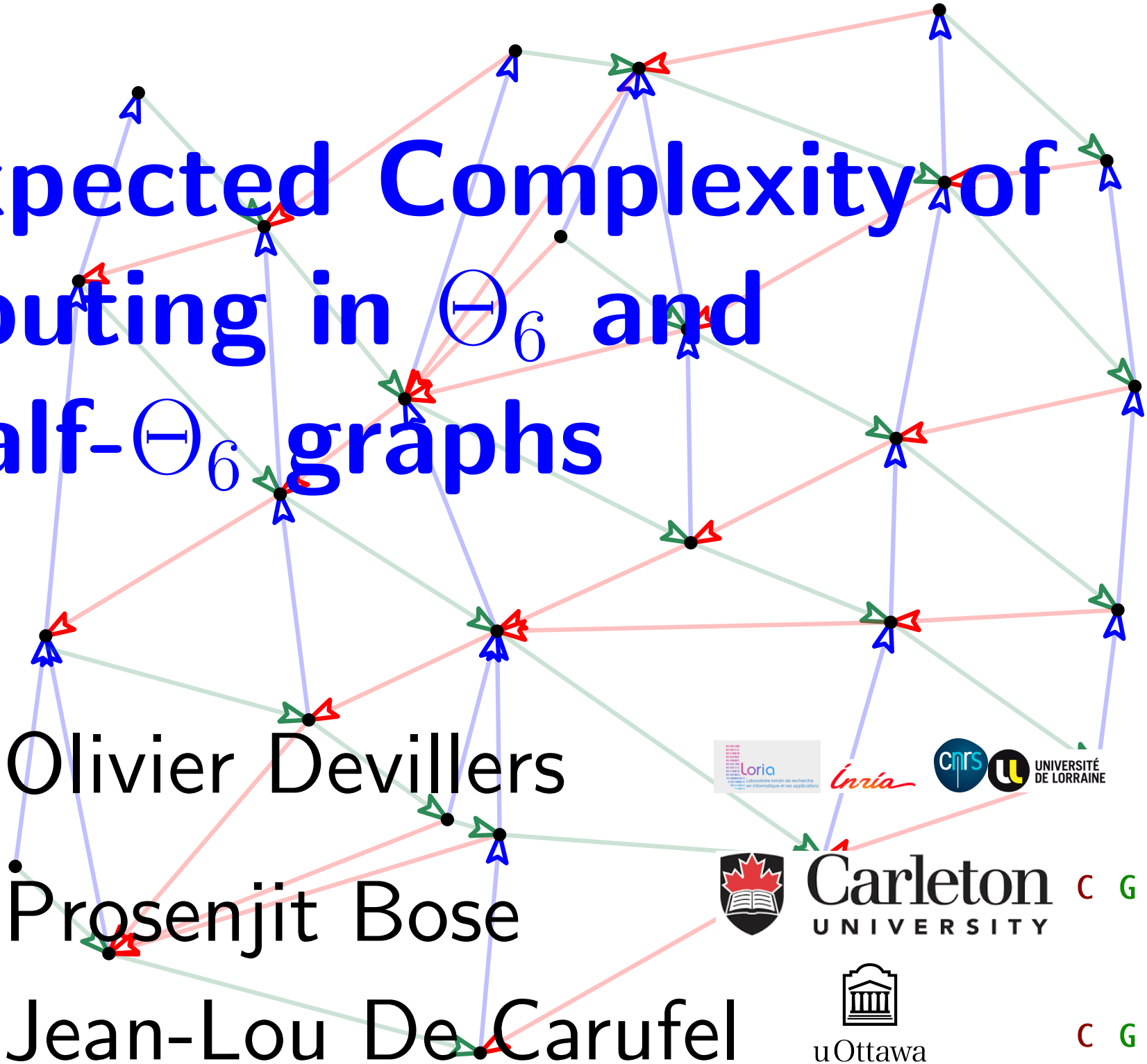
Expected Complexity of Routing in Θ_6 and Half-Θ_6 graphs

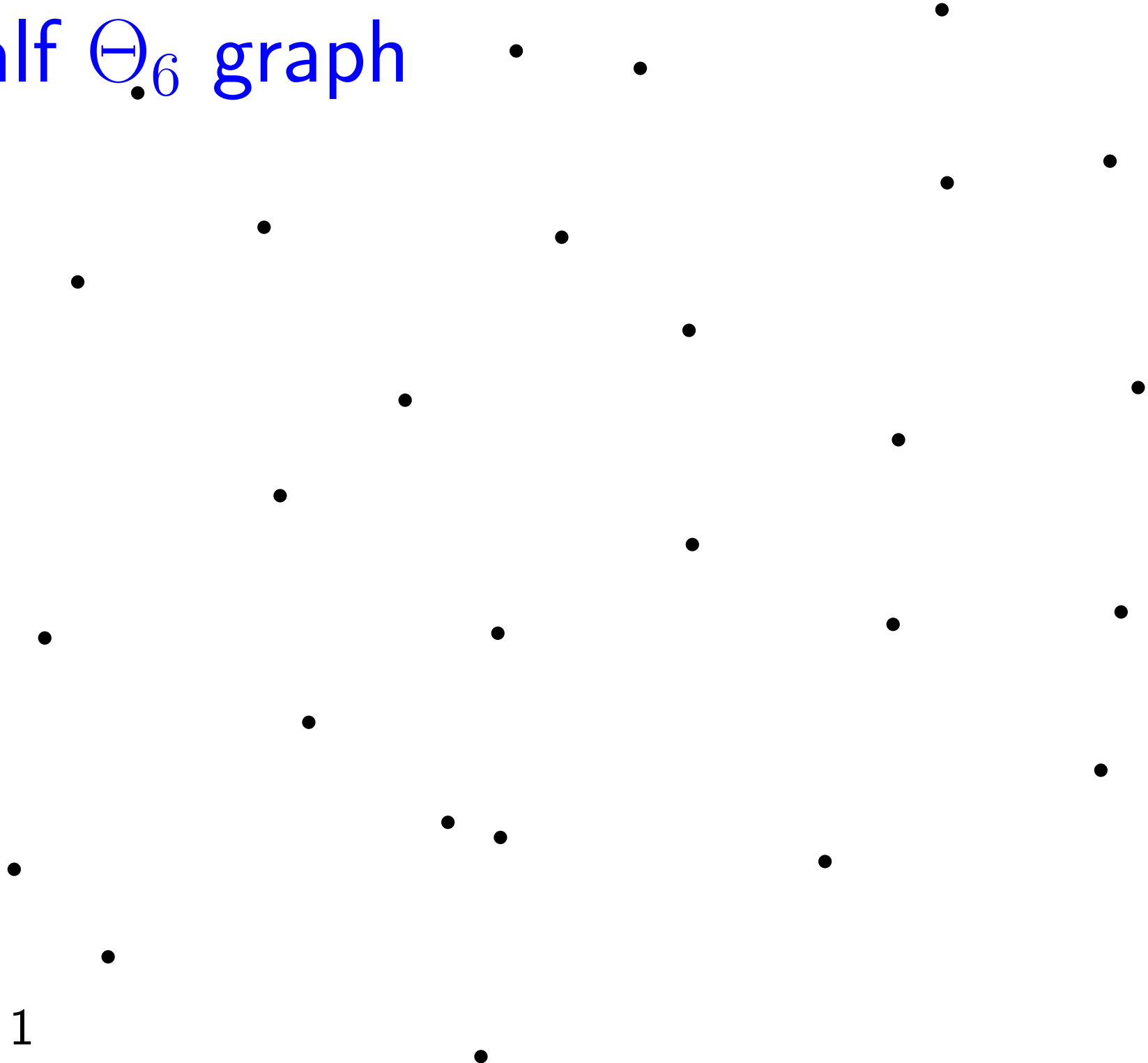


Olivier Devillers

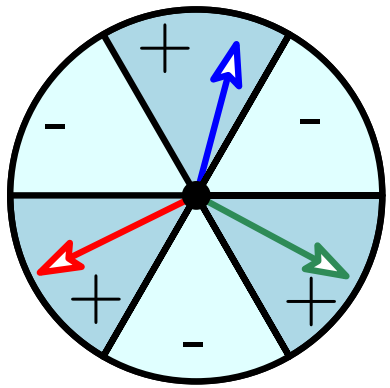
Prosenjit Bose

Jean-Lou De Carufel

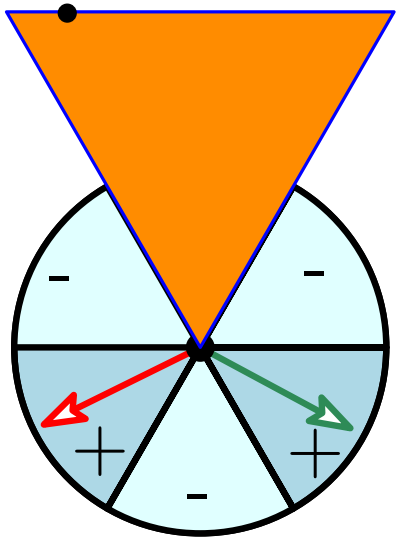
Half Θ_6 graph



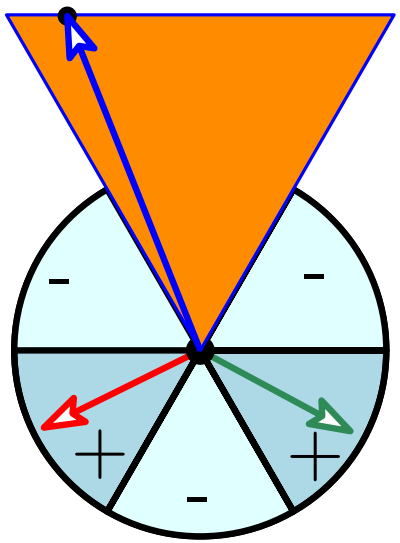
Half Θ_6 graph



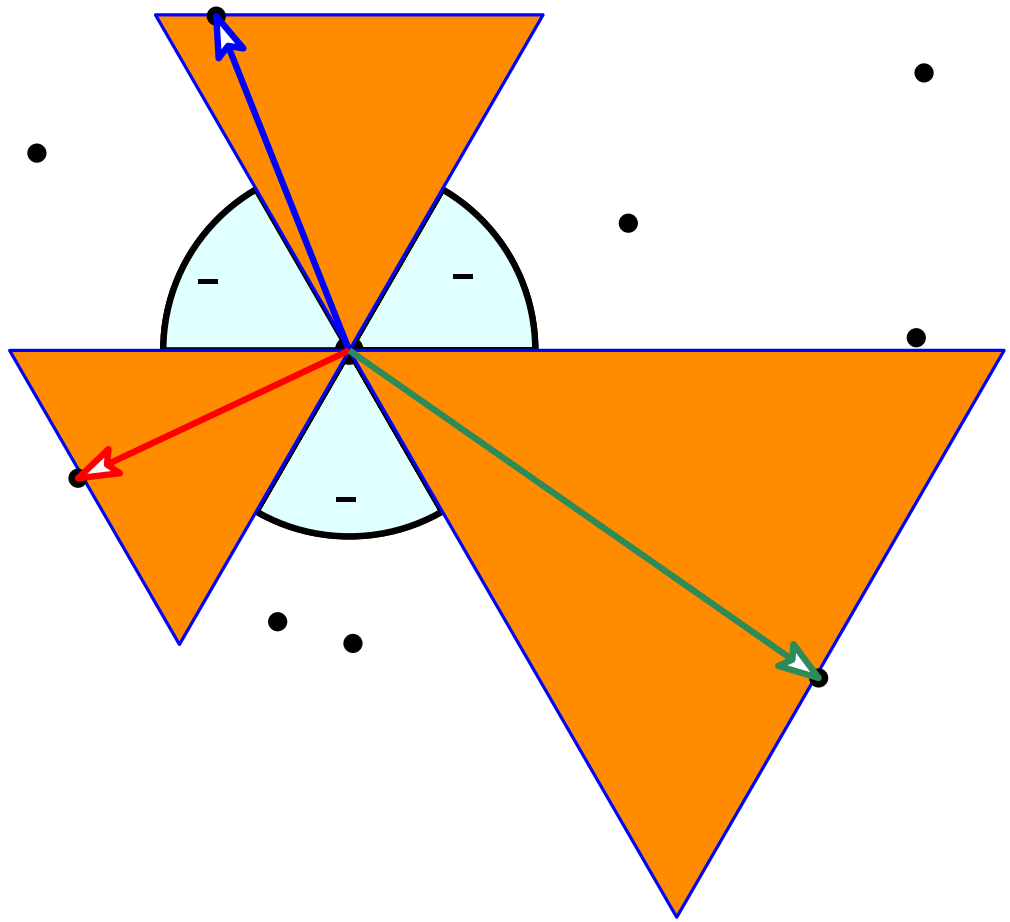
Half Θ_6 graph



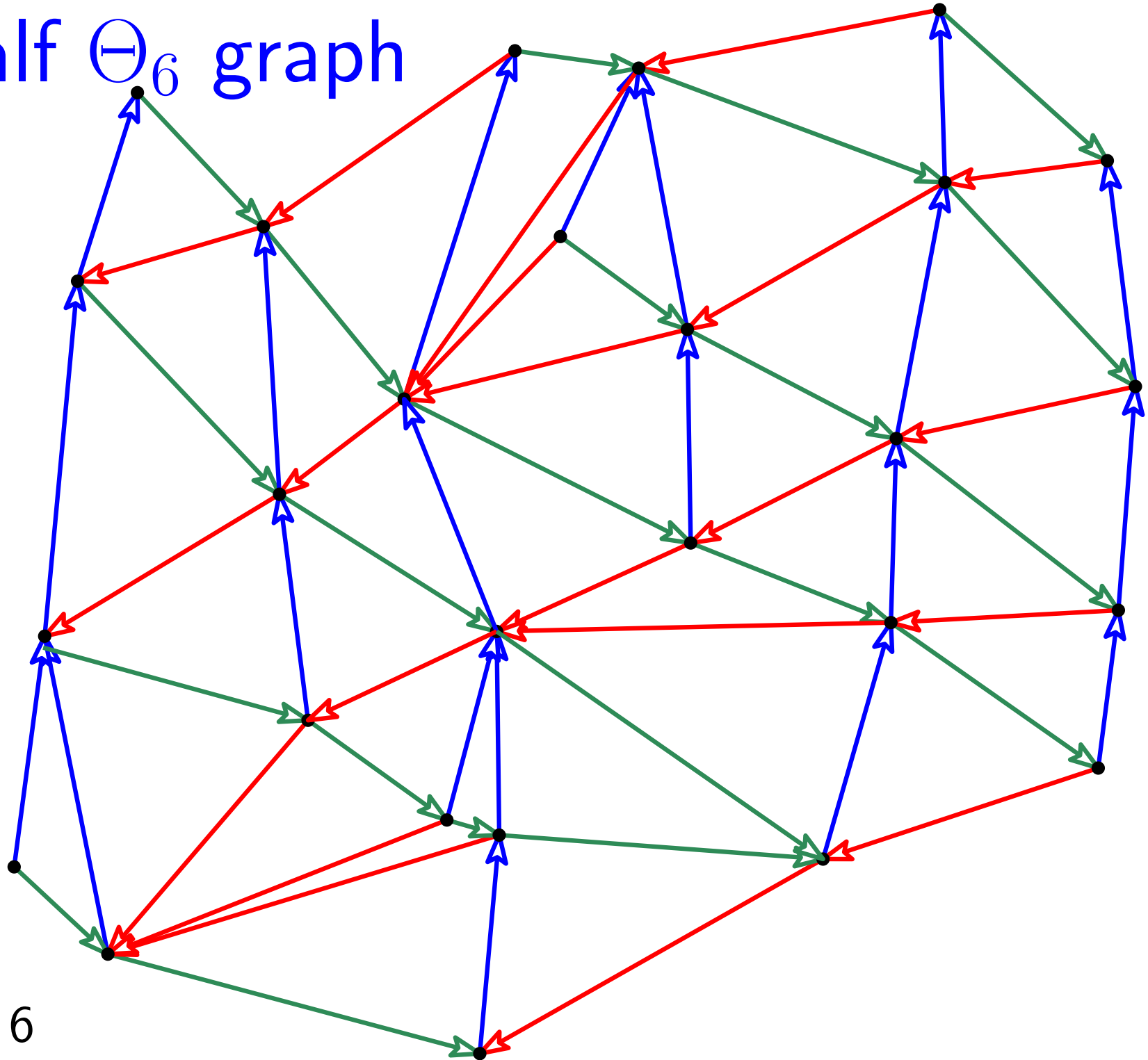
Half Θ_6 graph



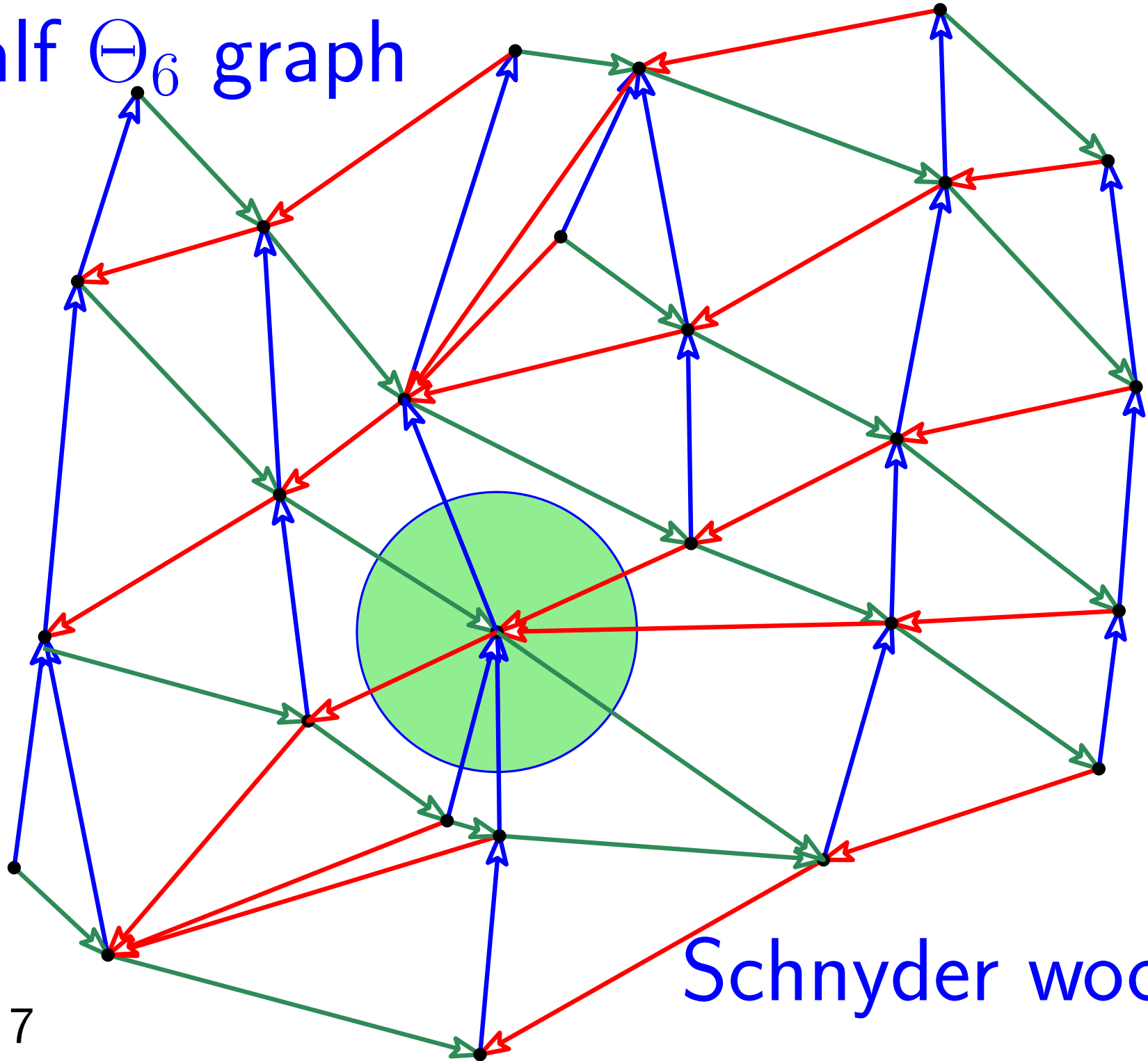
Half Θ_6 graph



Half Θ_6 graph

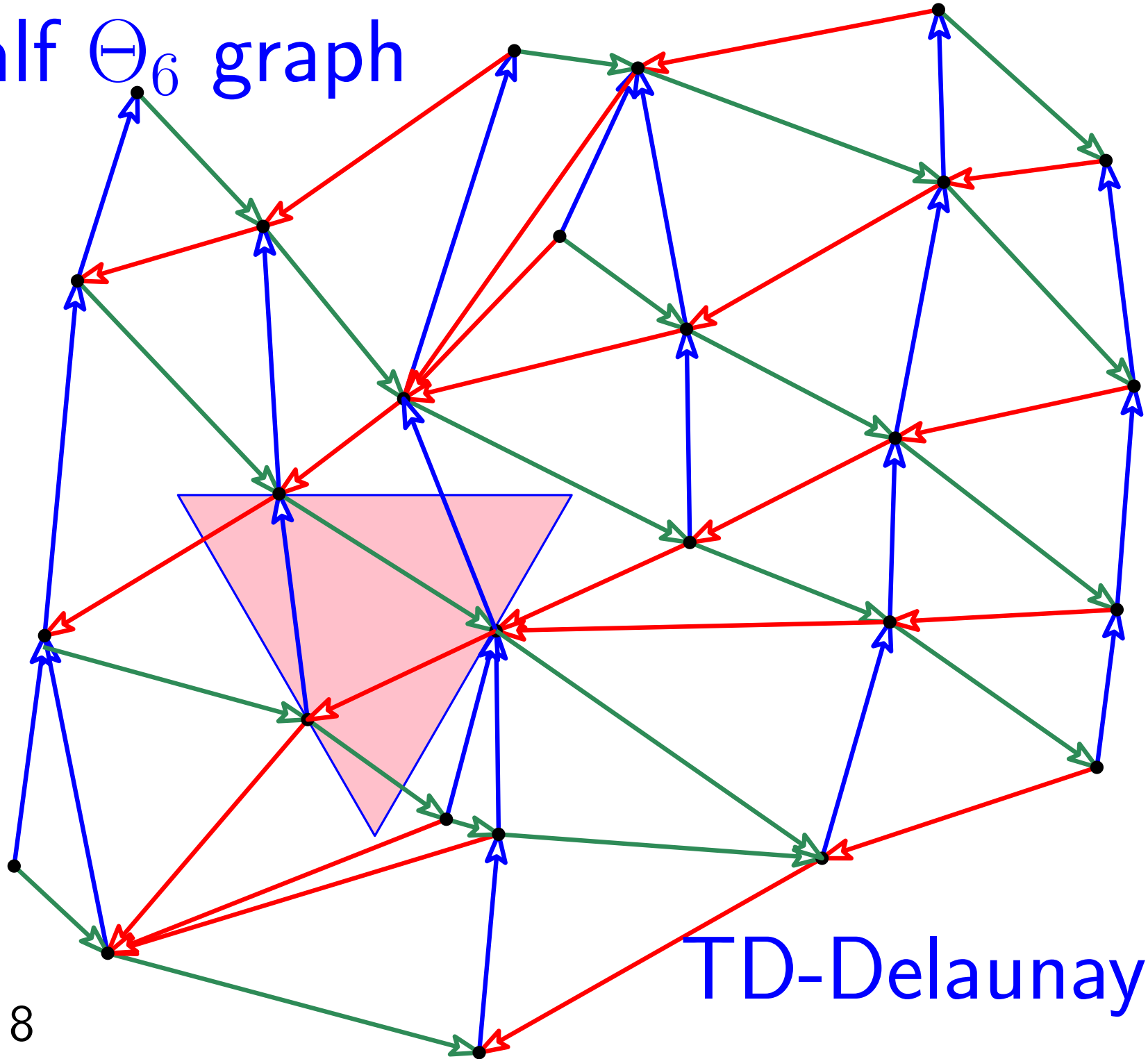


Half Θ_6 graph



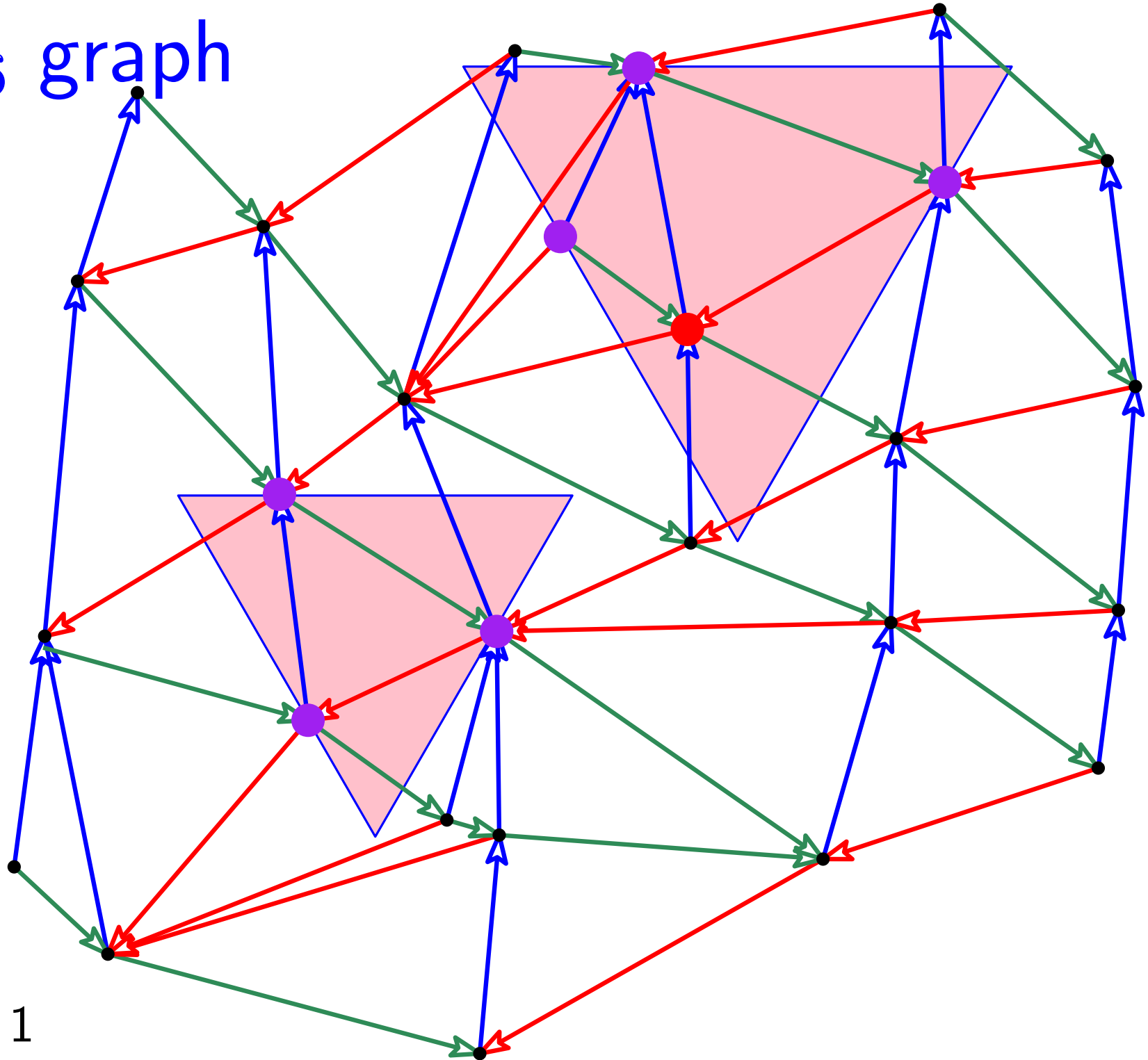
Schnyder wood

Half Θ_6 graph

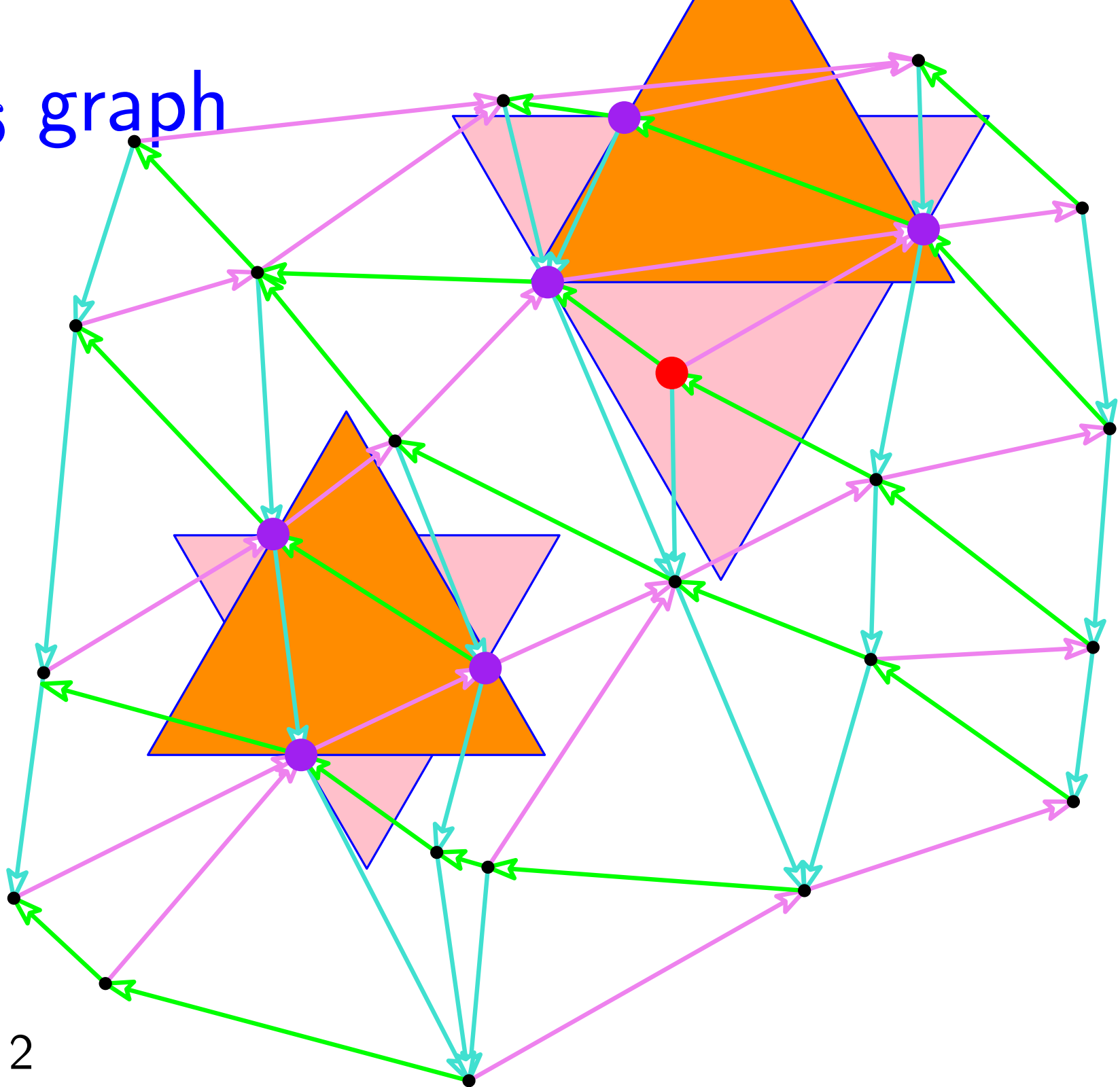


TD-Delaunay

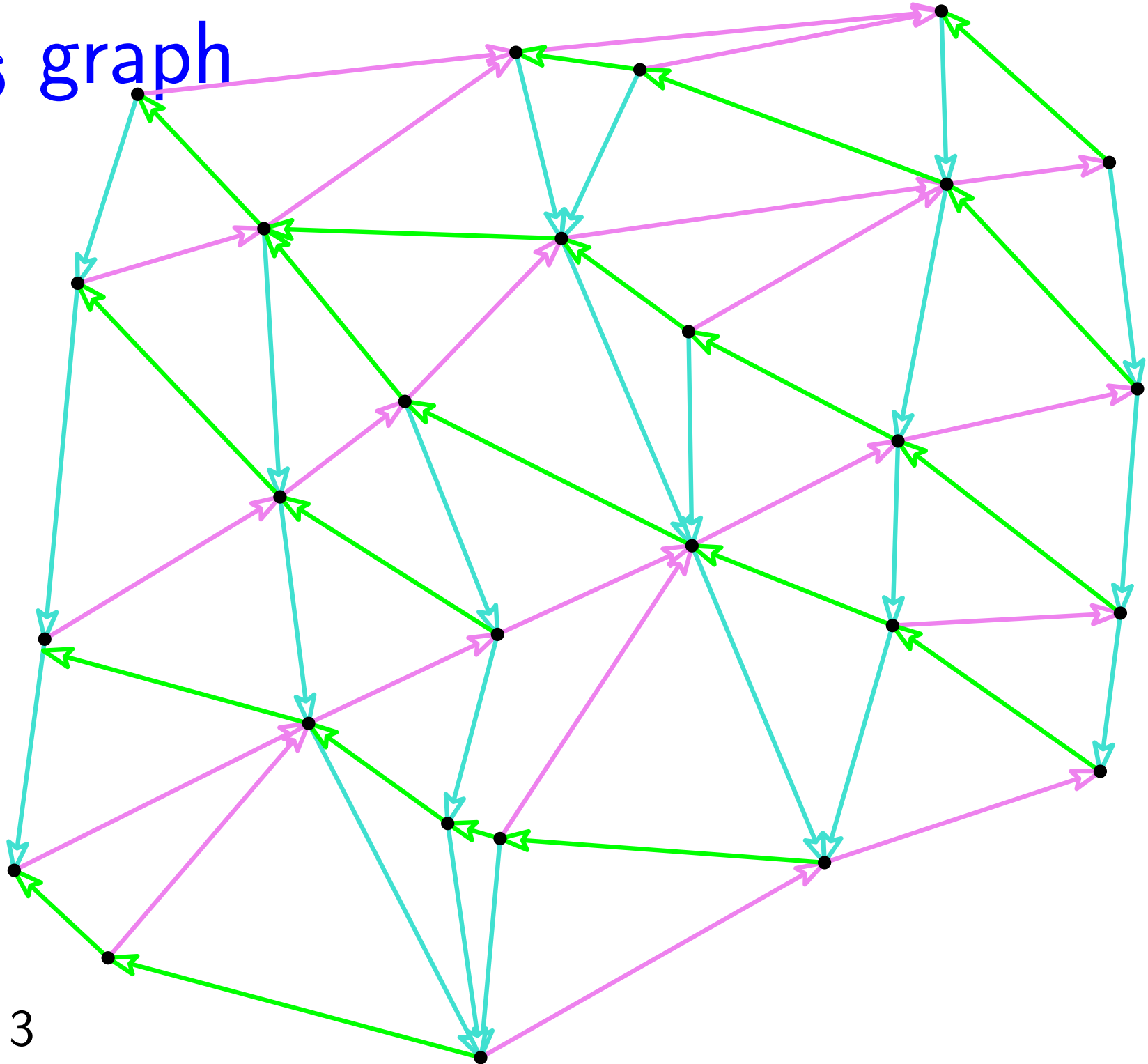
Θ_6 graph

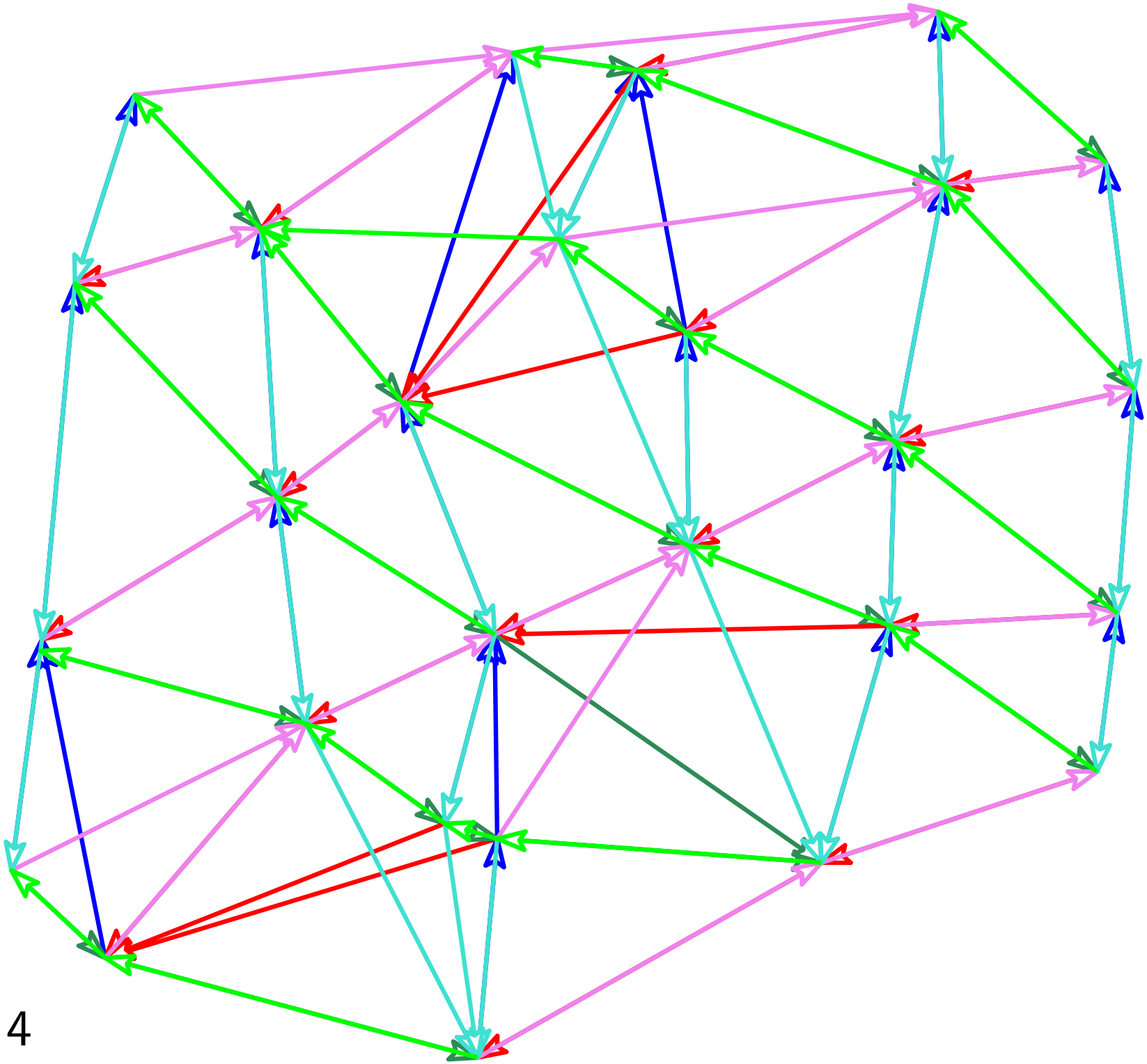


Θ_6 graph

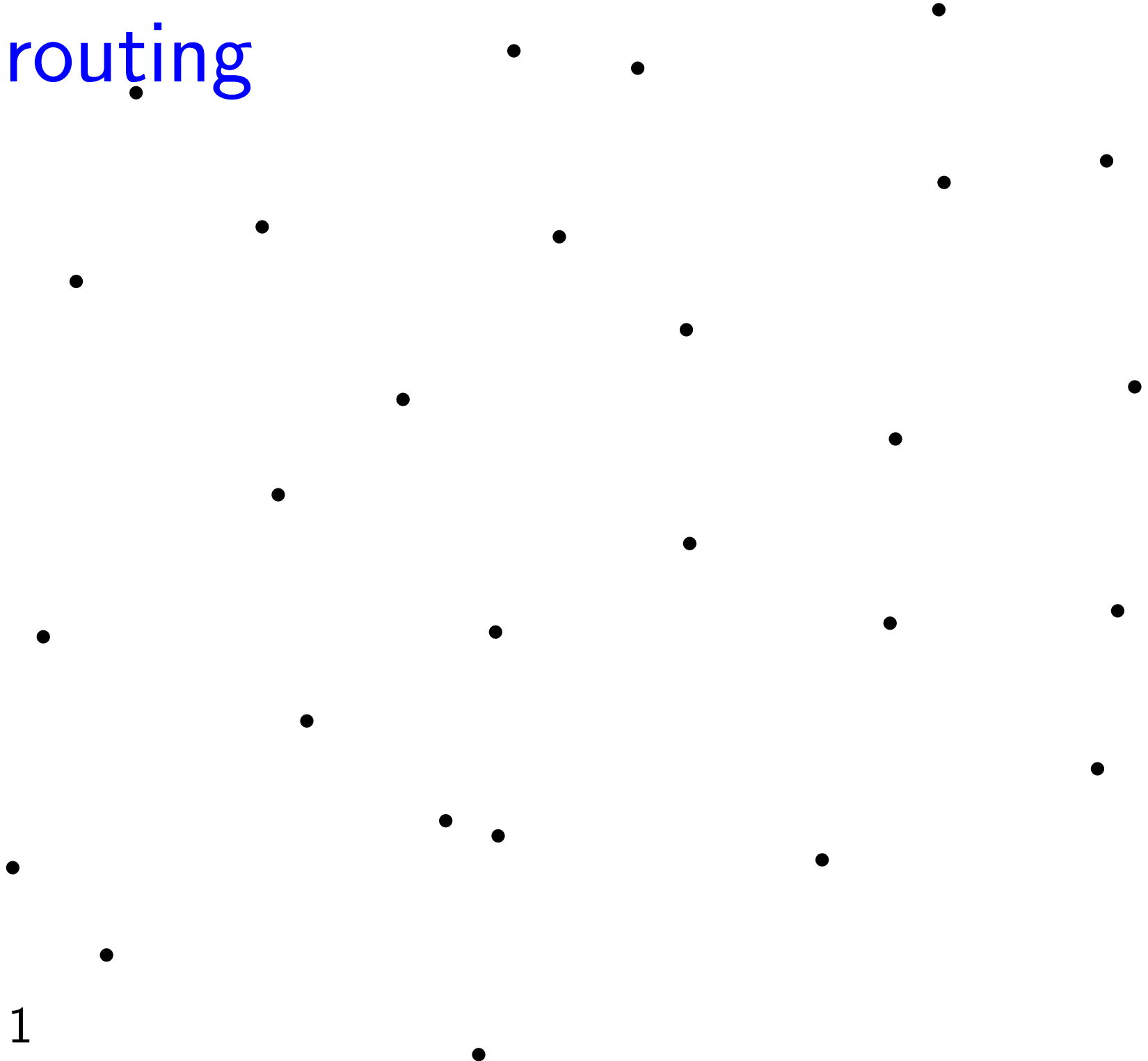


Θ_6 graph



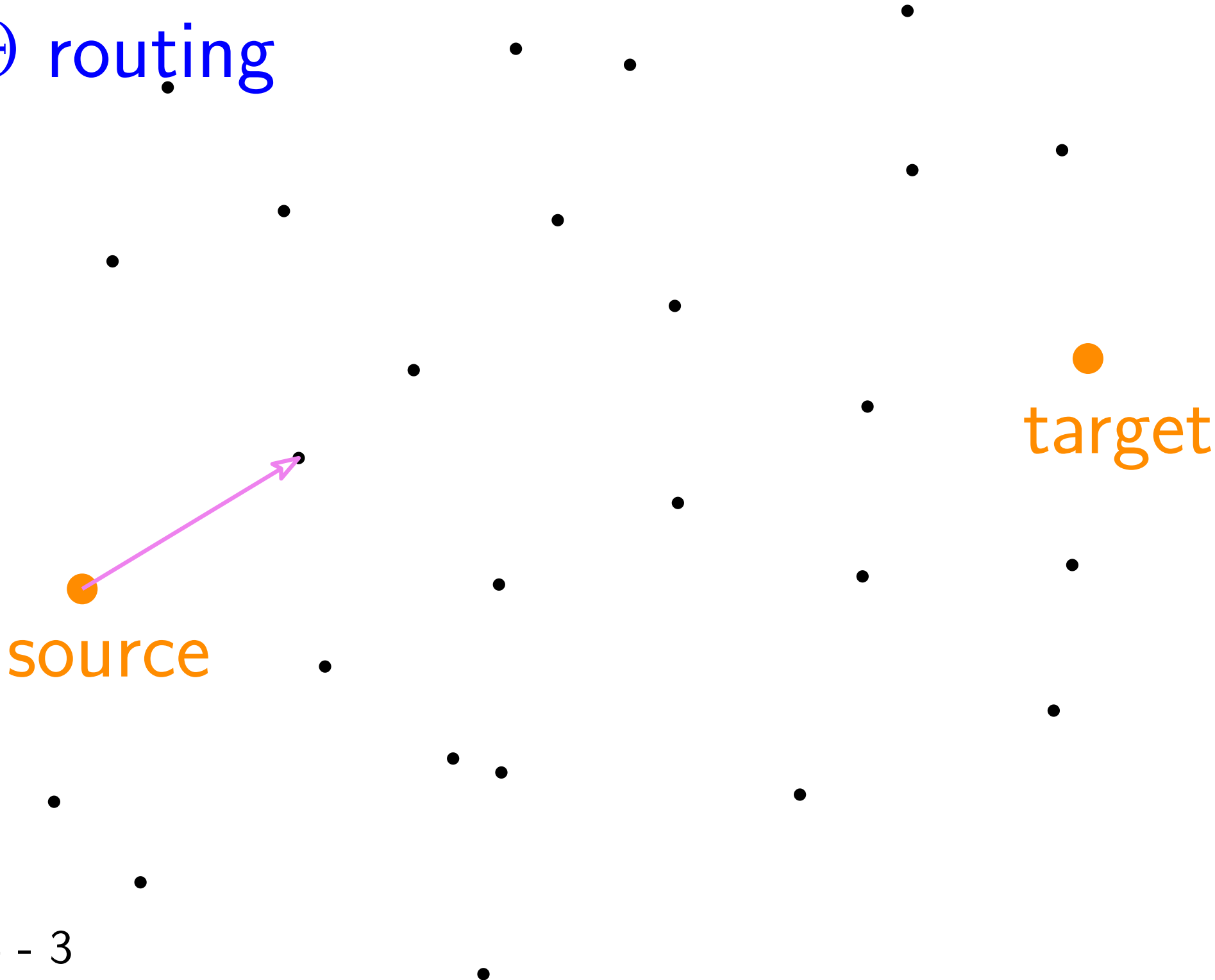


Θ routing

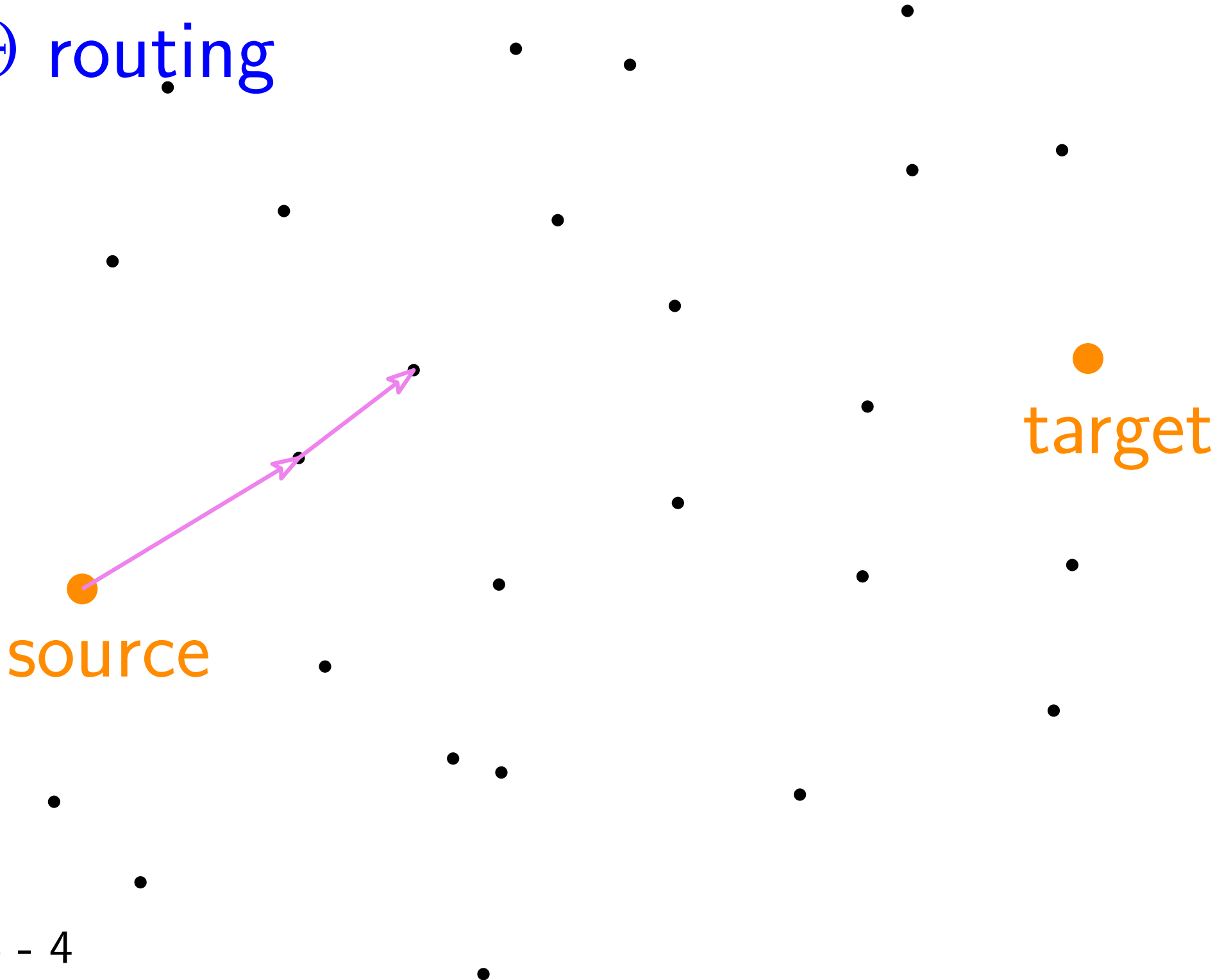



Θ routing
target
source

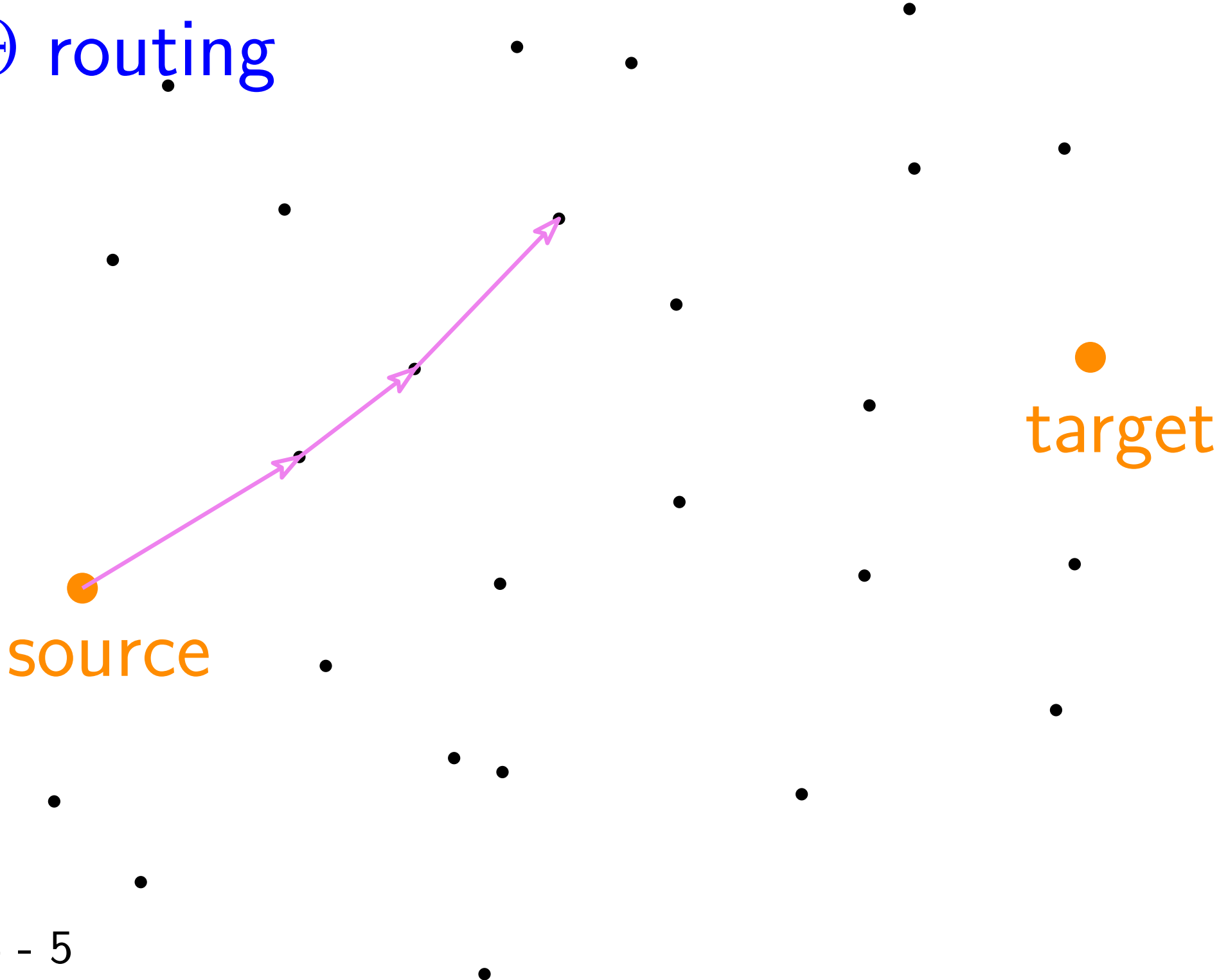
Θ routing



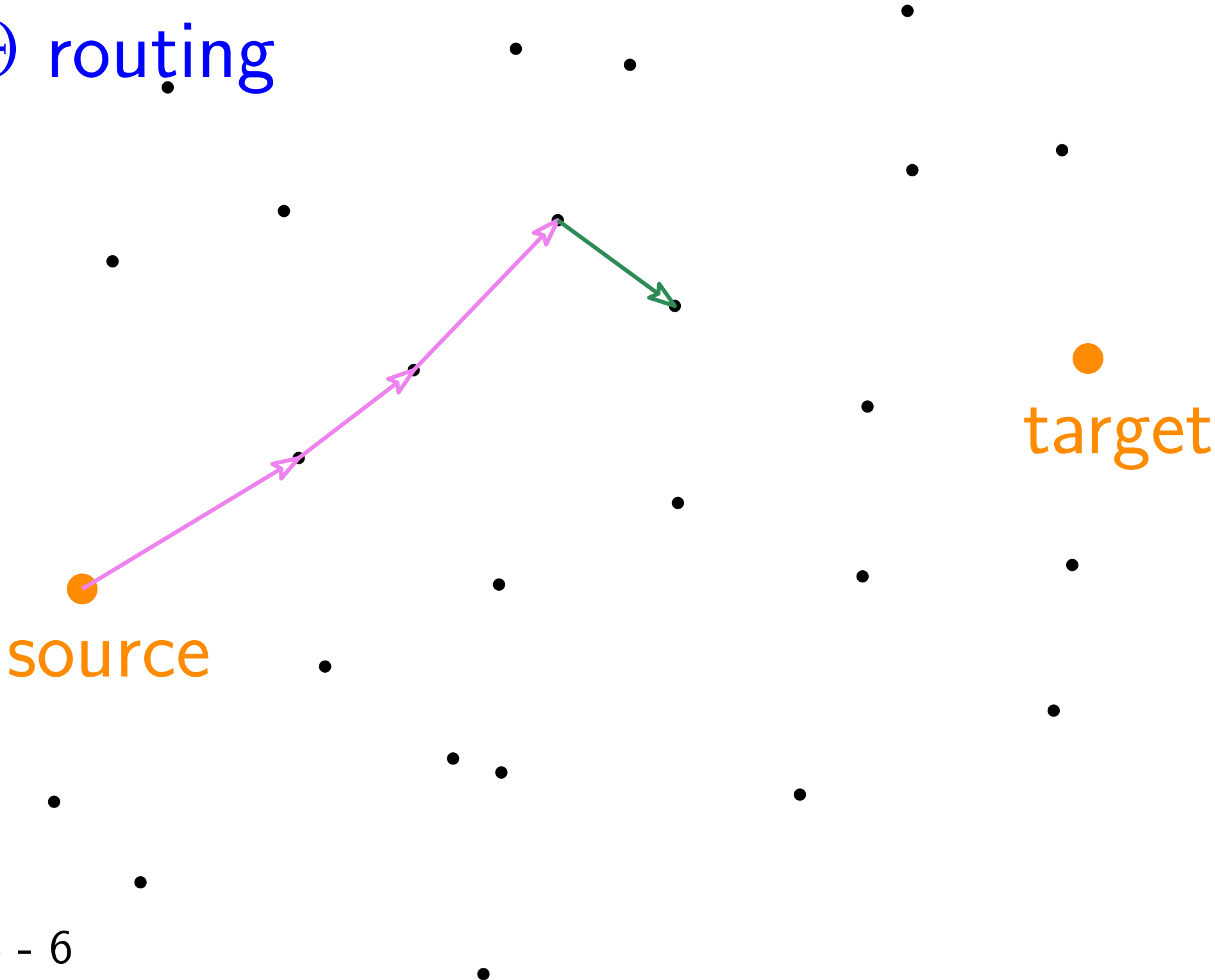
Θ routing



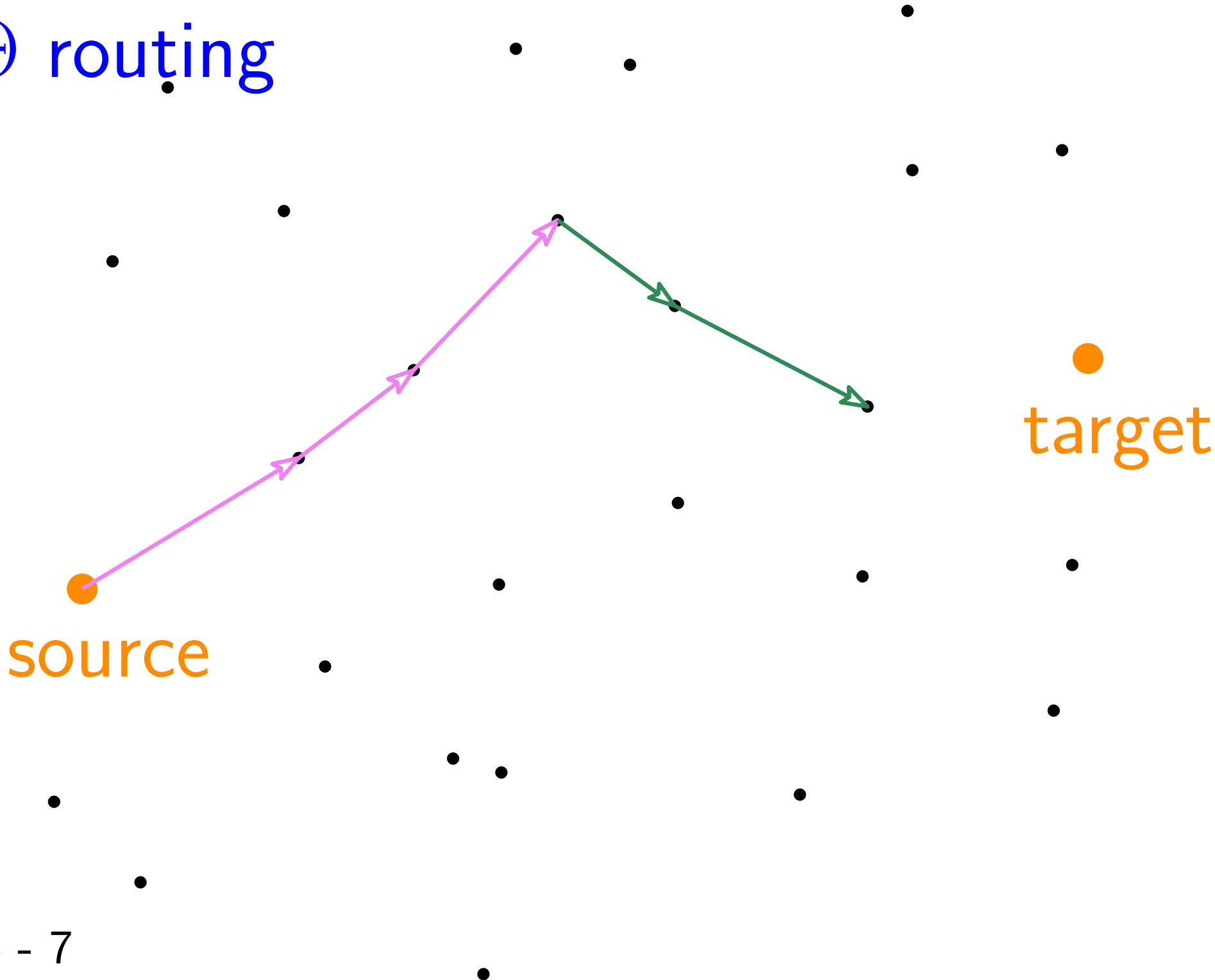
Θ routing



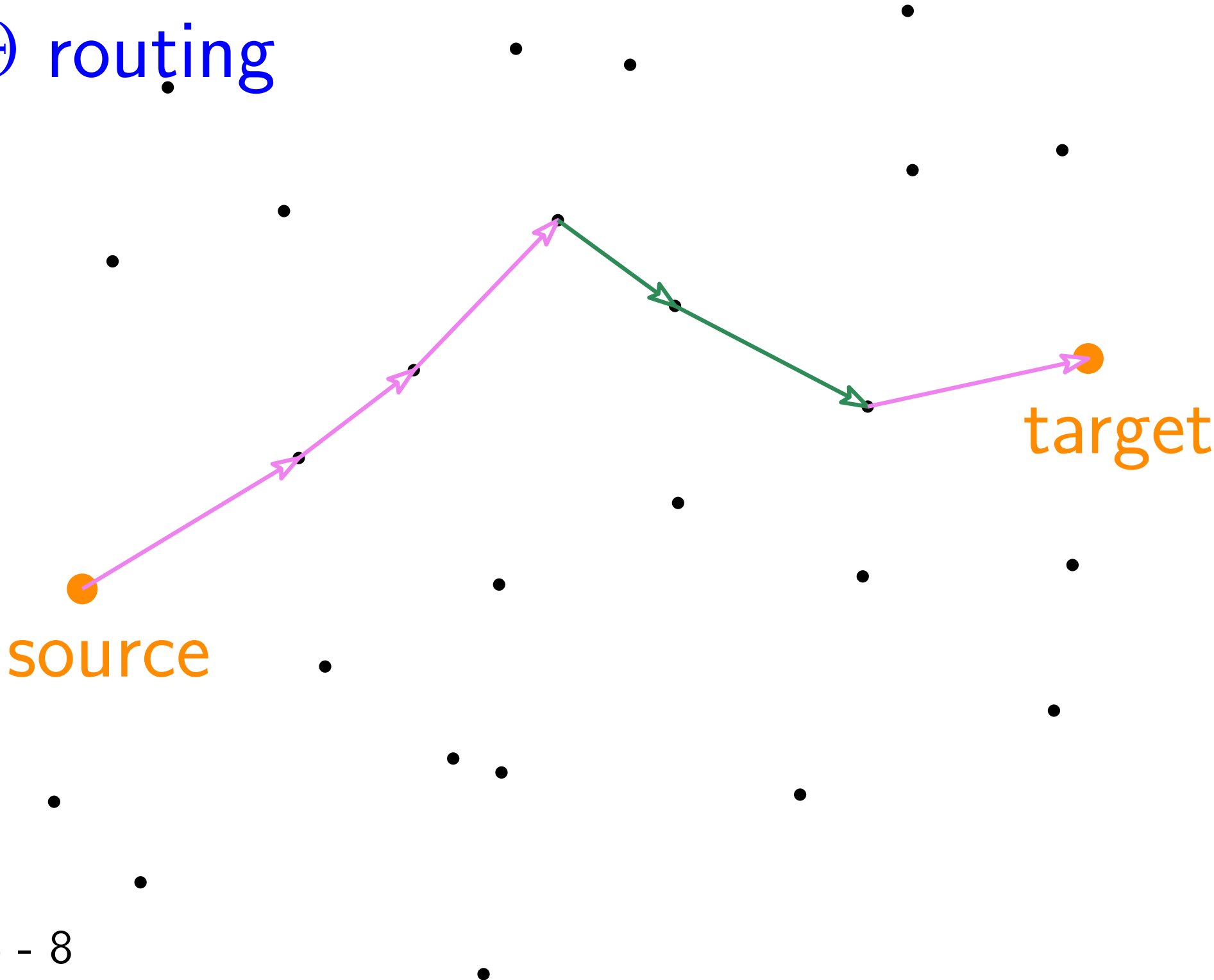
Θ routing



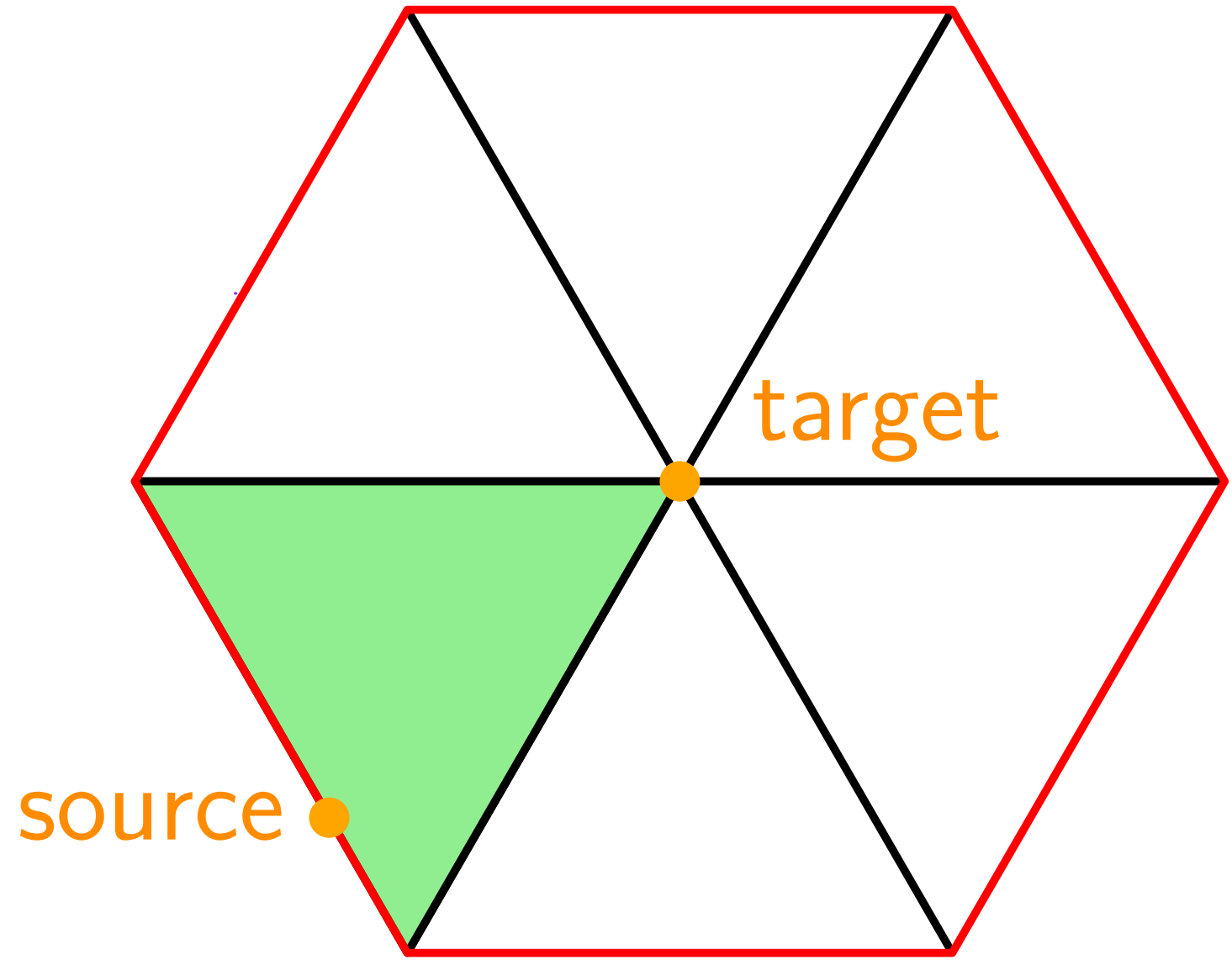
Θ routing



Θ routing



Θ routing



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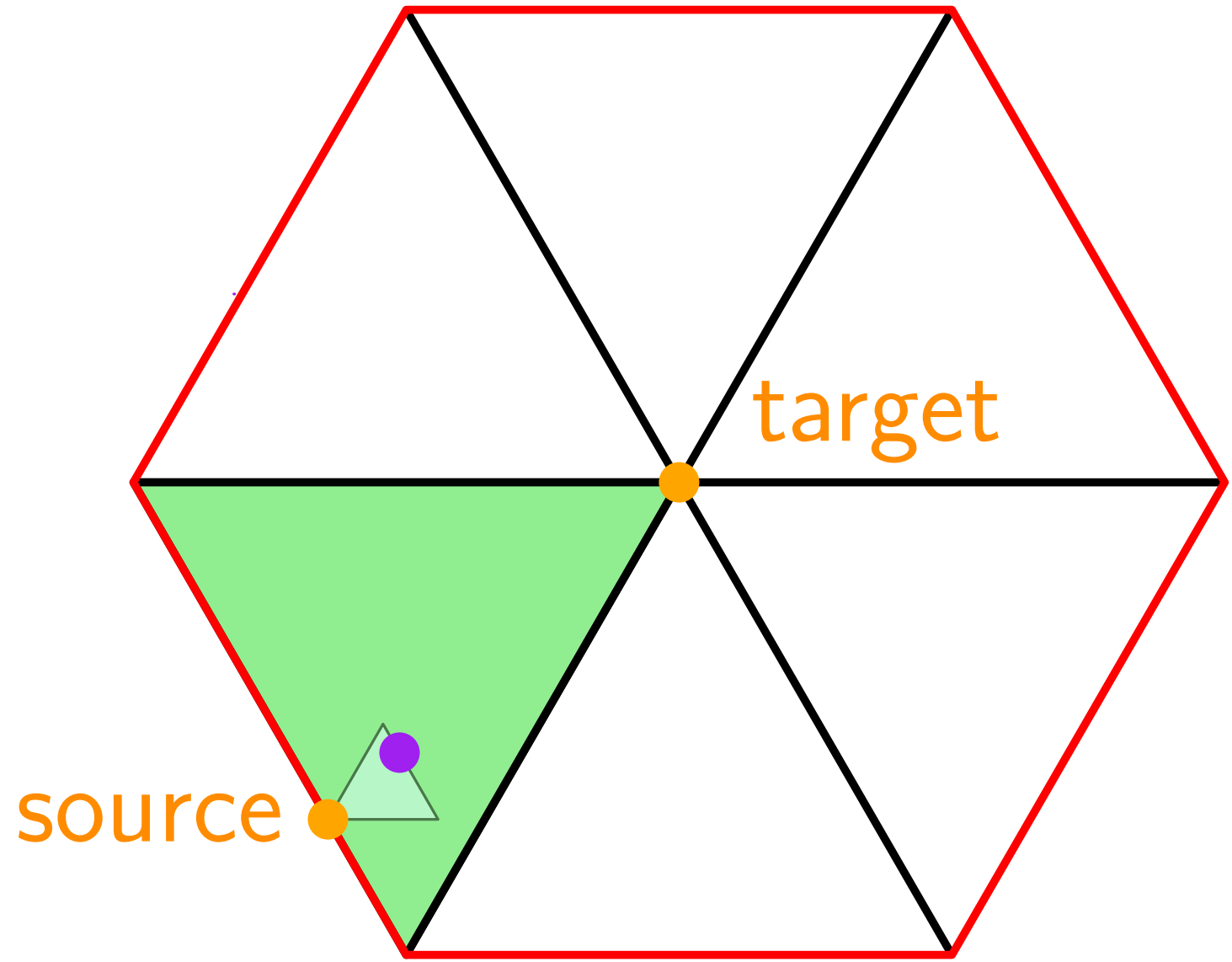
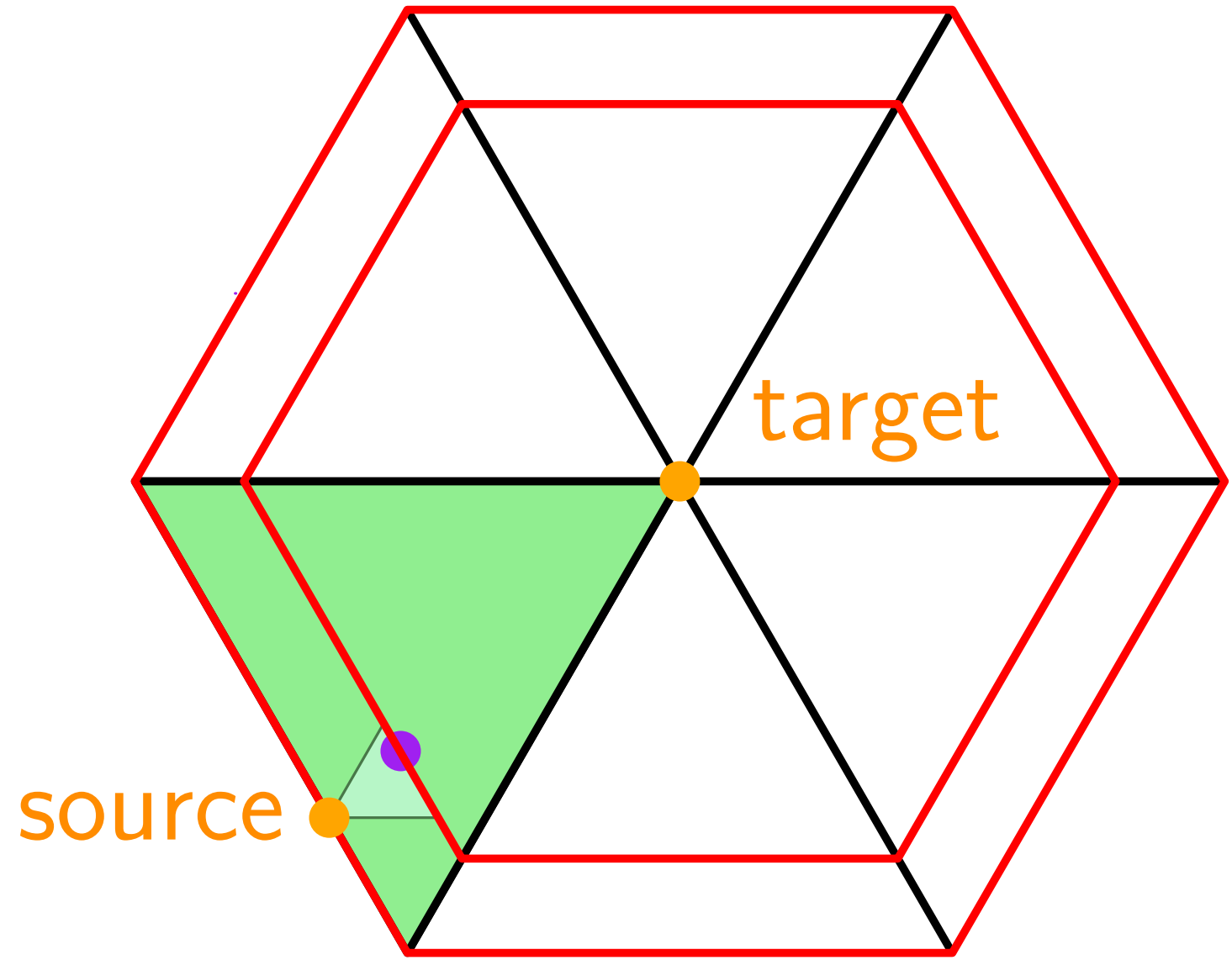


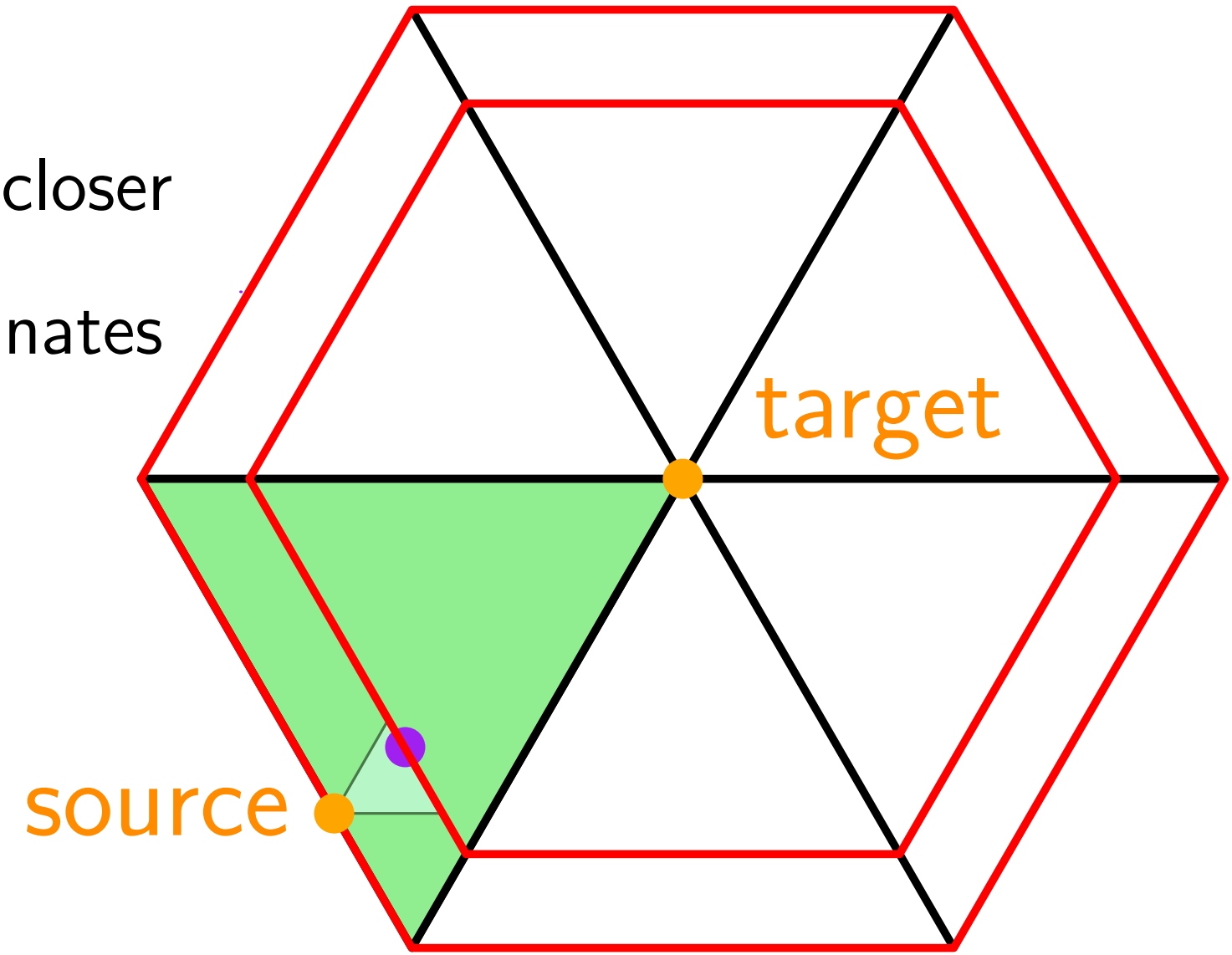
Θ routing



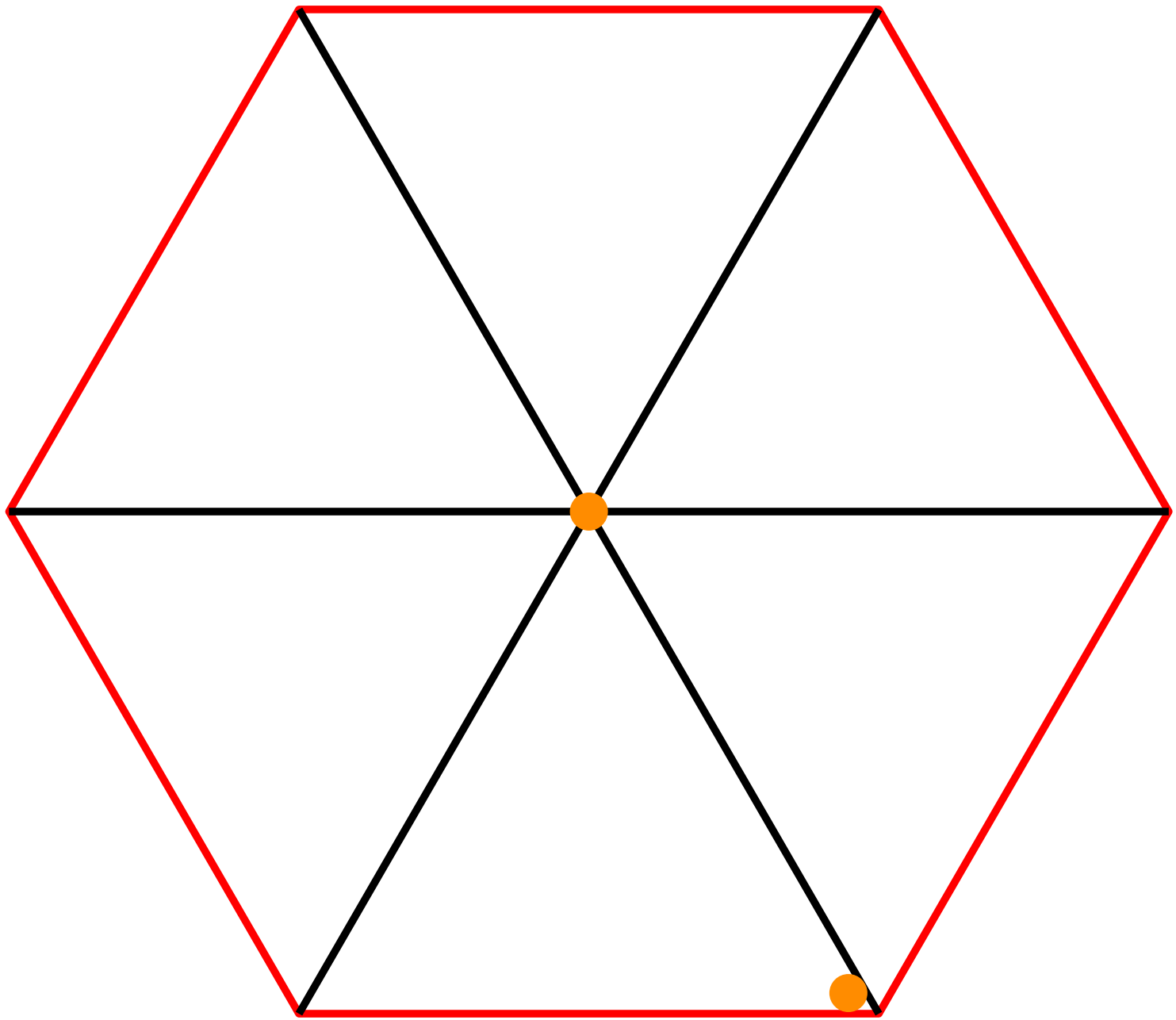
Θ routing

Next point is closer

Routing terminates



Θ routing



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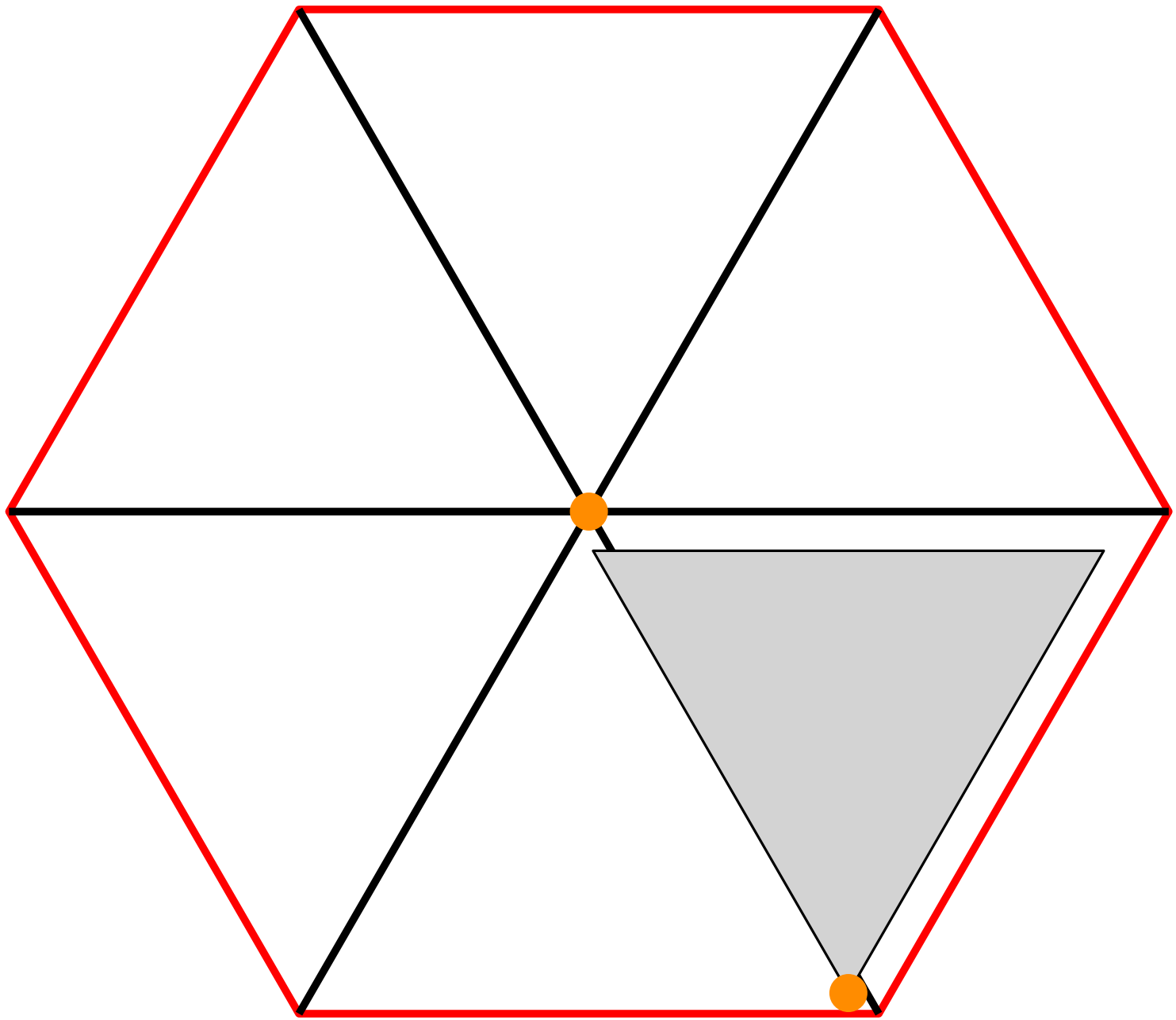
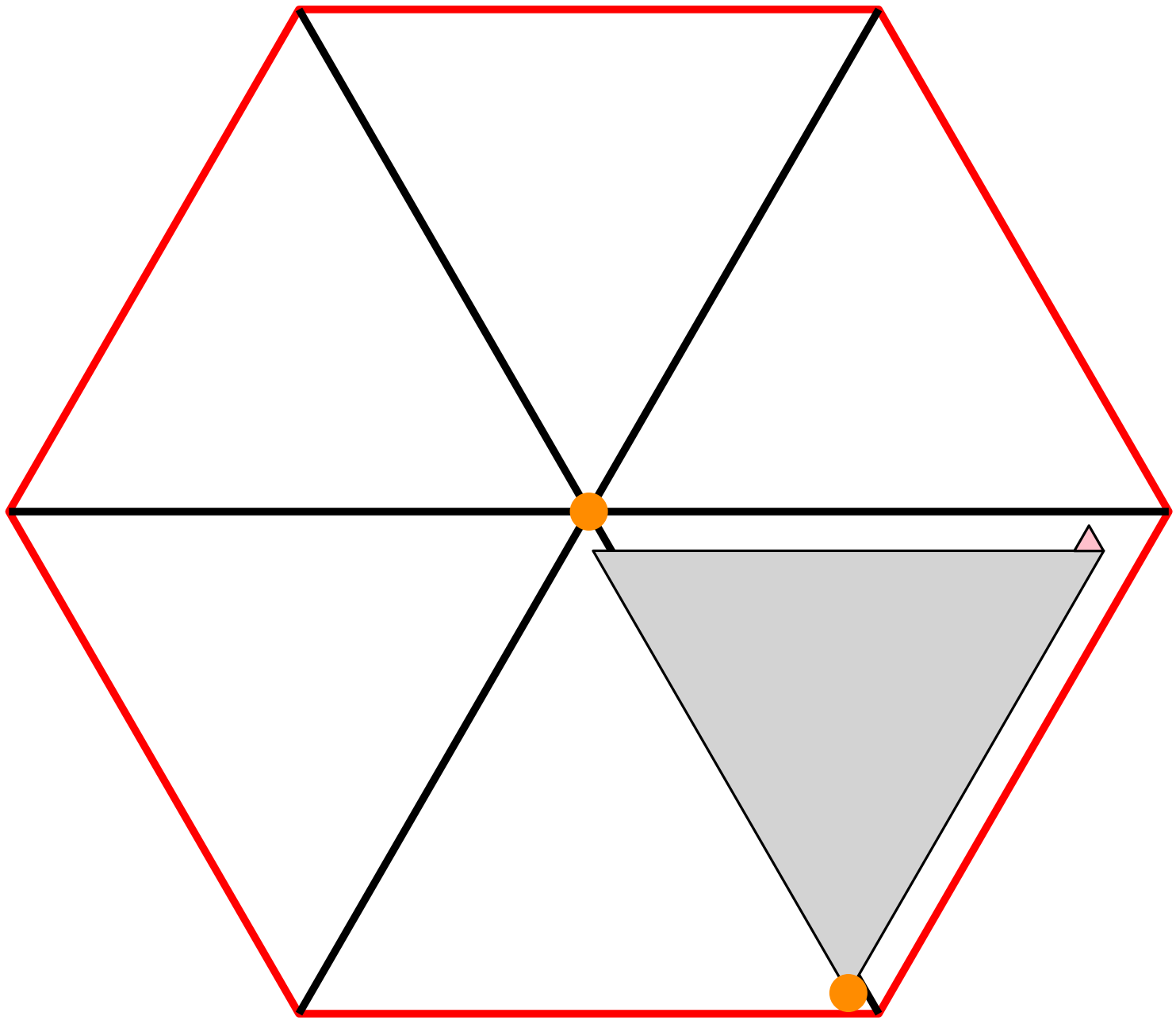
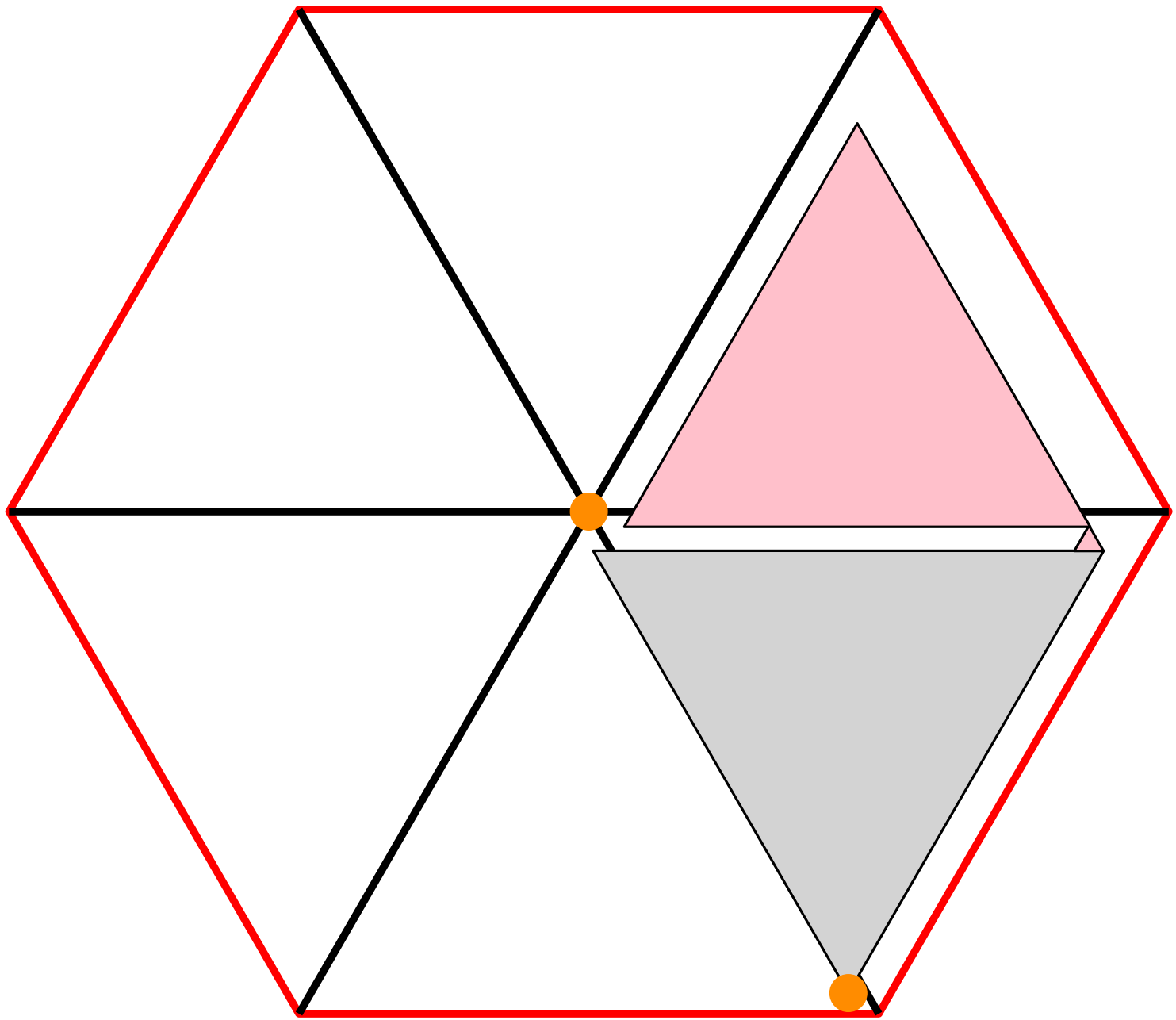


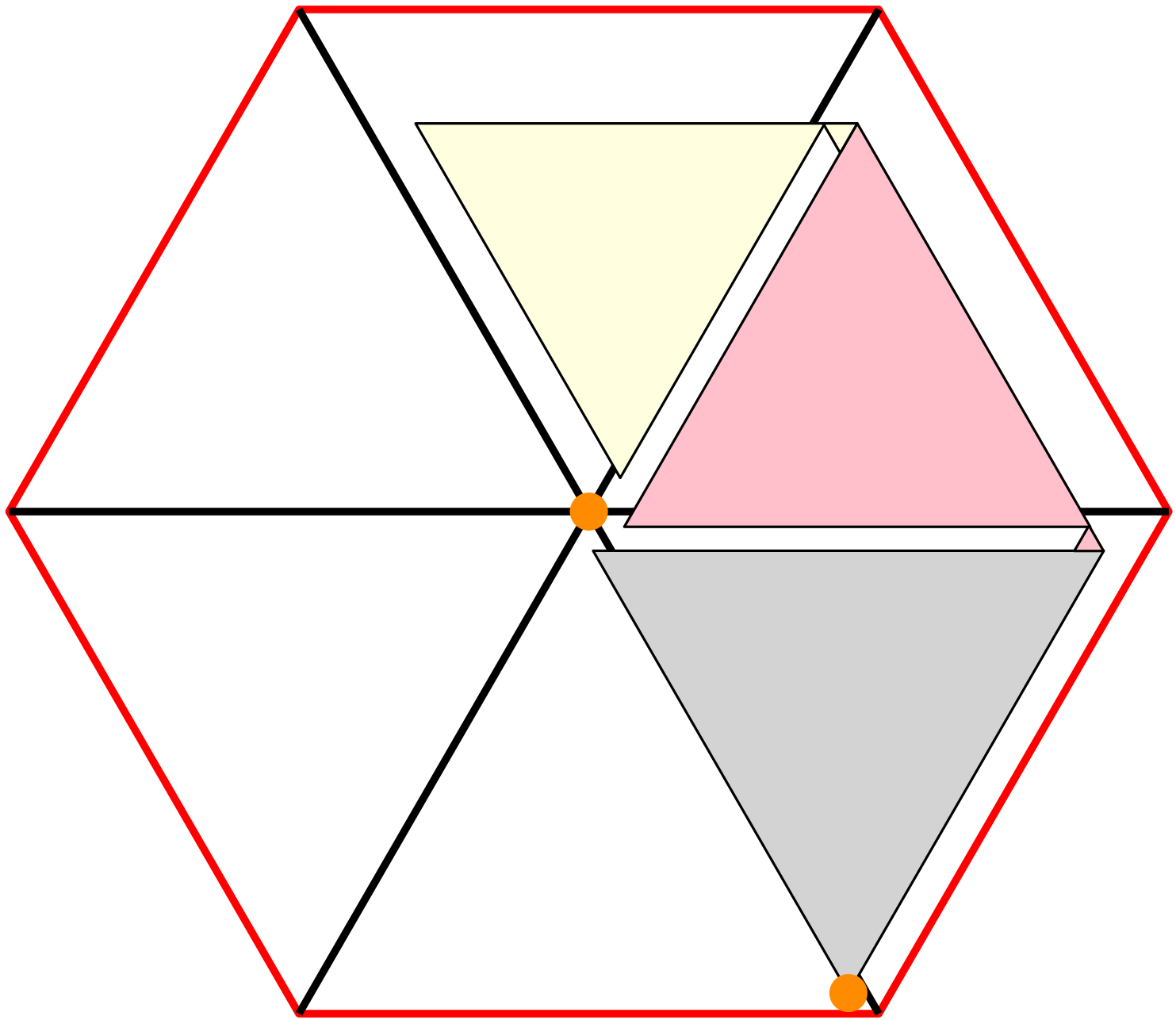
Θ routing



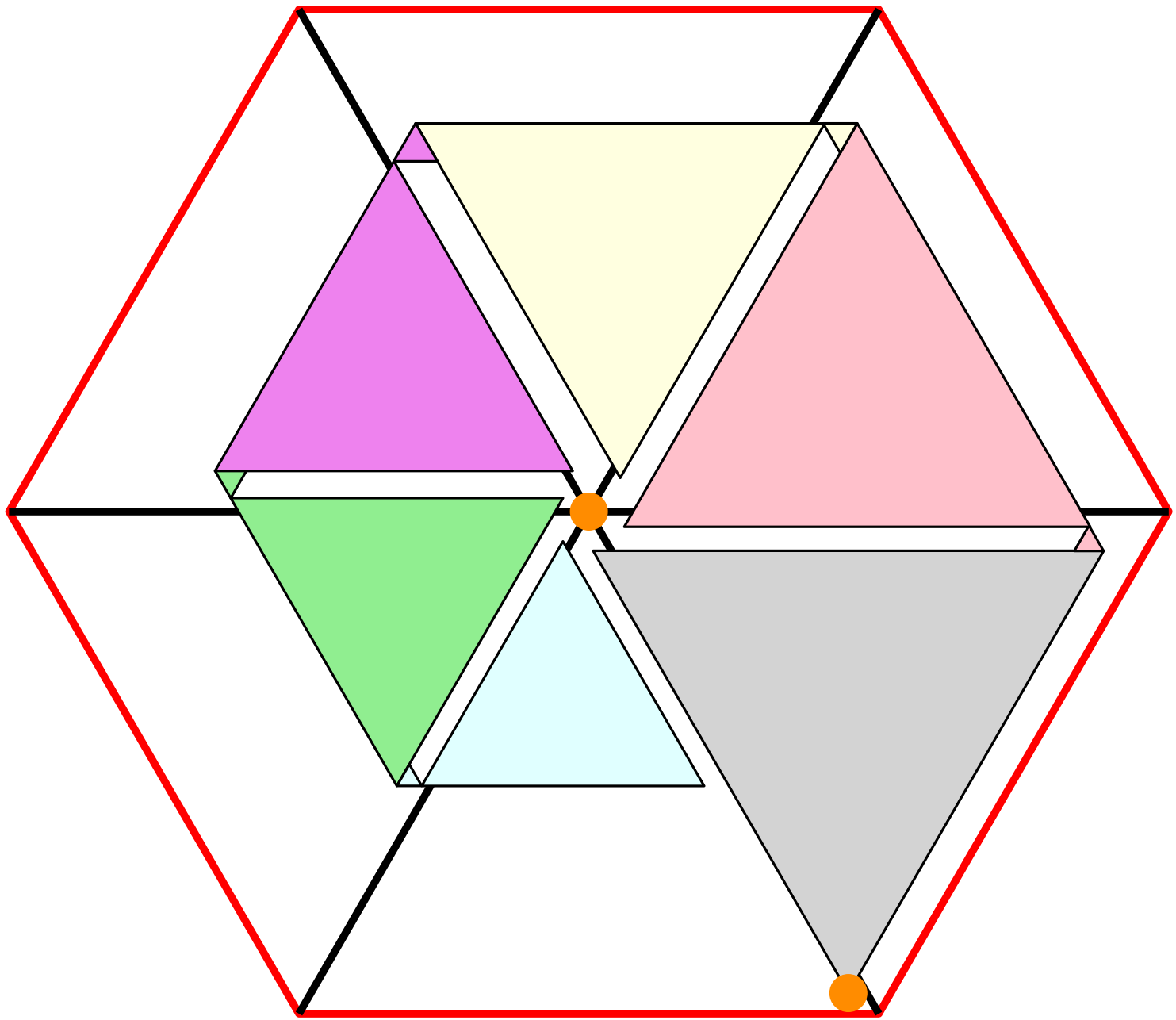
Θ routing



Θ routing

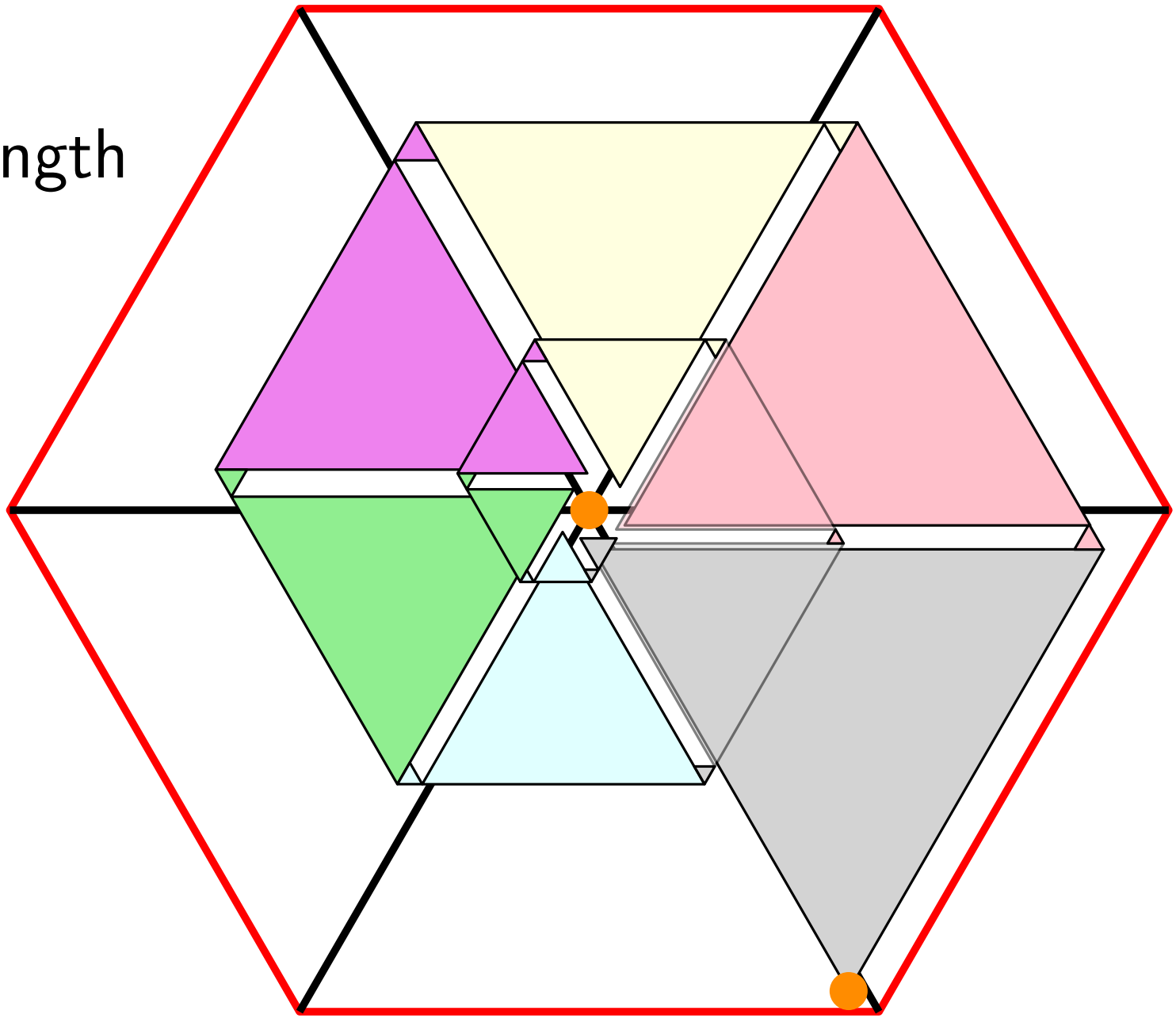


Θ routing



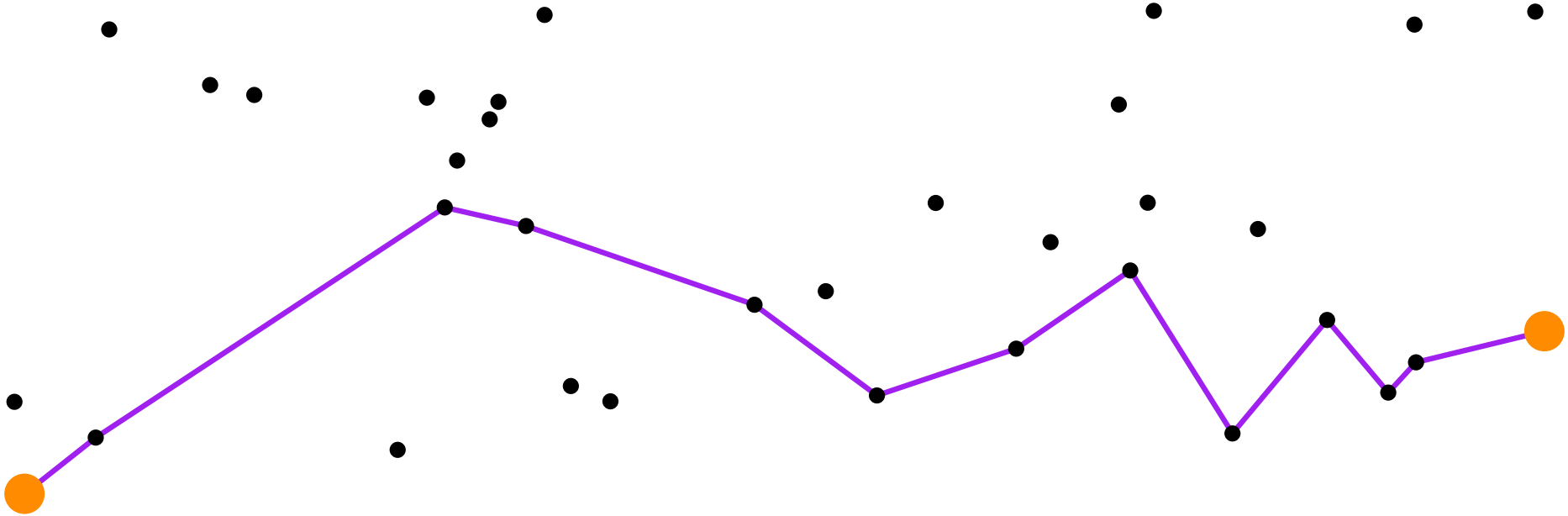
Θ routing

Unbounded length



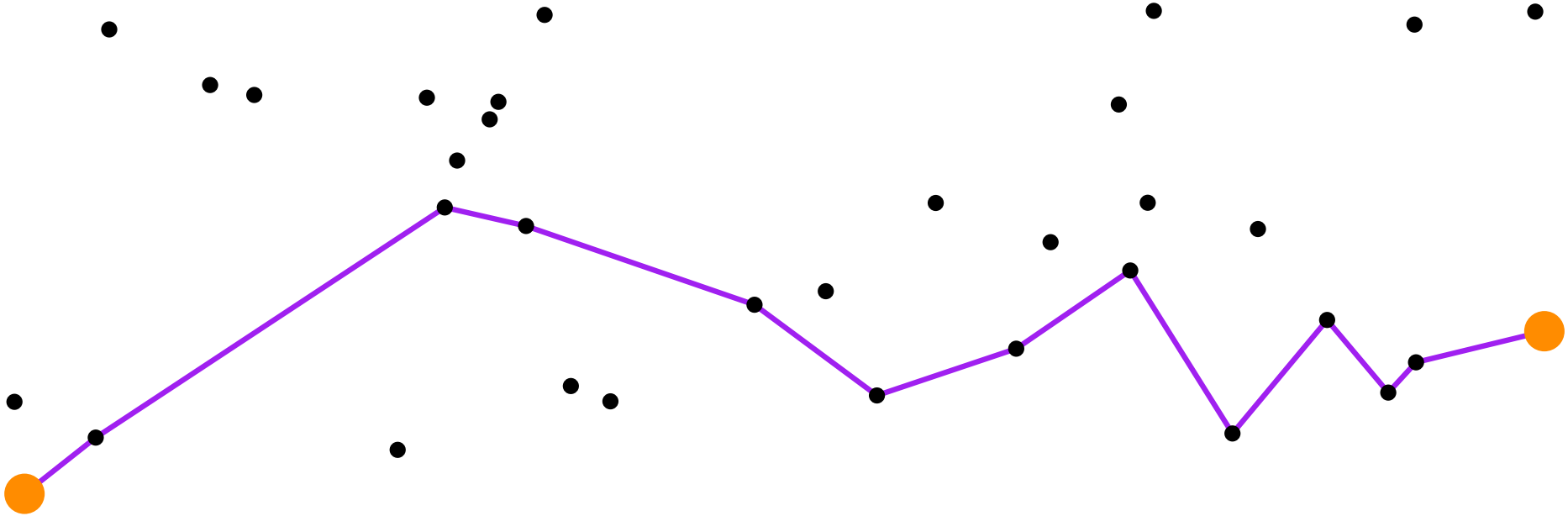
Θ routing

Expected length for Poisson distribution



Θ routing

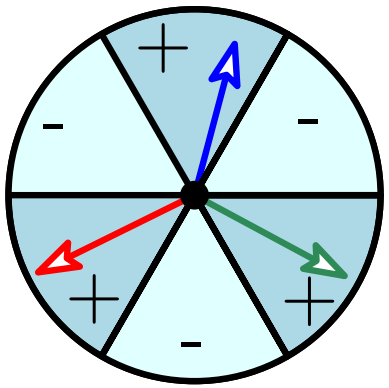
Expected length for Poisson distribution



$$\leq \tfrac{1}{6}(3\ln 3 + 4) \simeq 1.216$$

[Bonichon Marcket]

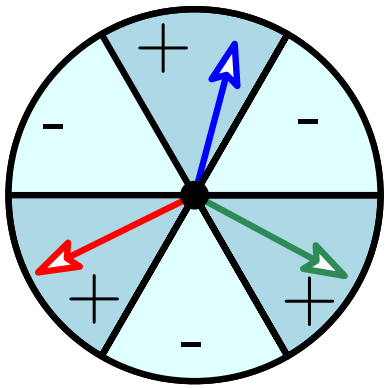
Routing in half Θ_6



Positive routing

Negative routing

Routing in half Θ_6



Positive routing ≤ 2

Negative routing $\leq \frac{5}{\sqrt{3}} \simeq 2.89$

[Bose, Fagerberg, van Renssen, Verdonschot]

Our contribution

Two tools: forward routing and side routing

New negative routing algorithms

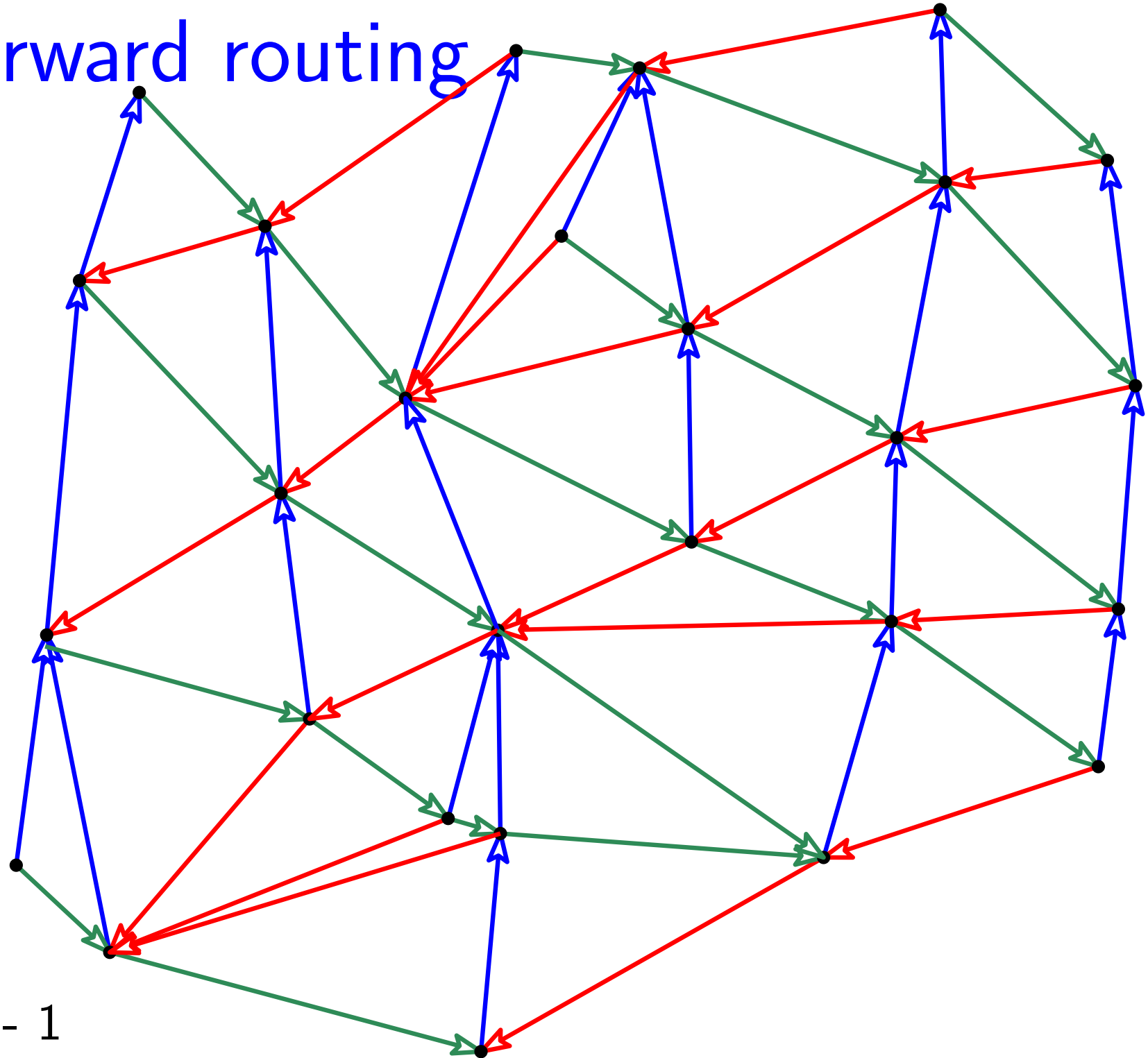
same worst-case bound

Probabilistic analysis for Poisson distribution

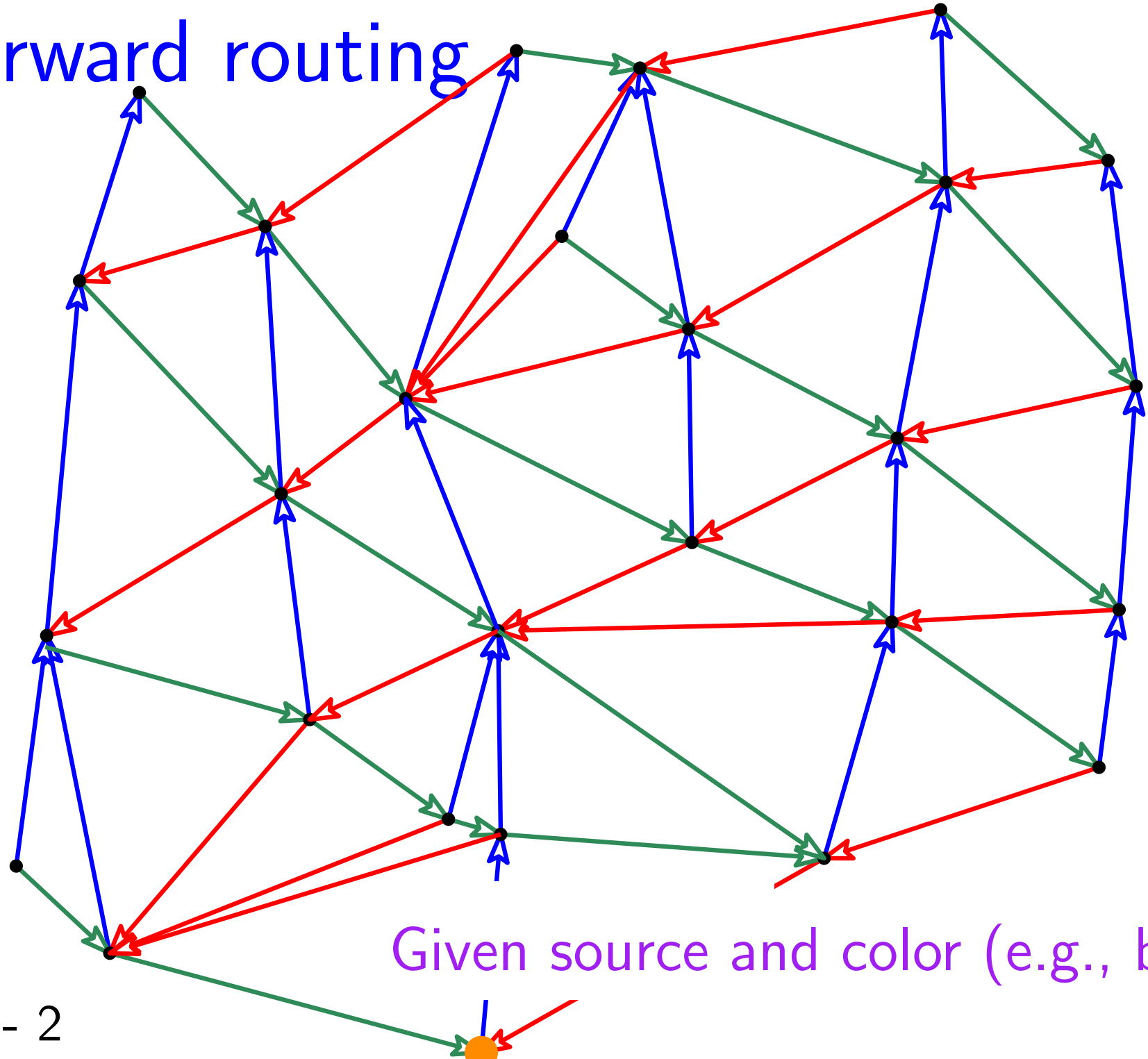
positive routing

new negative routings

Forward routing

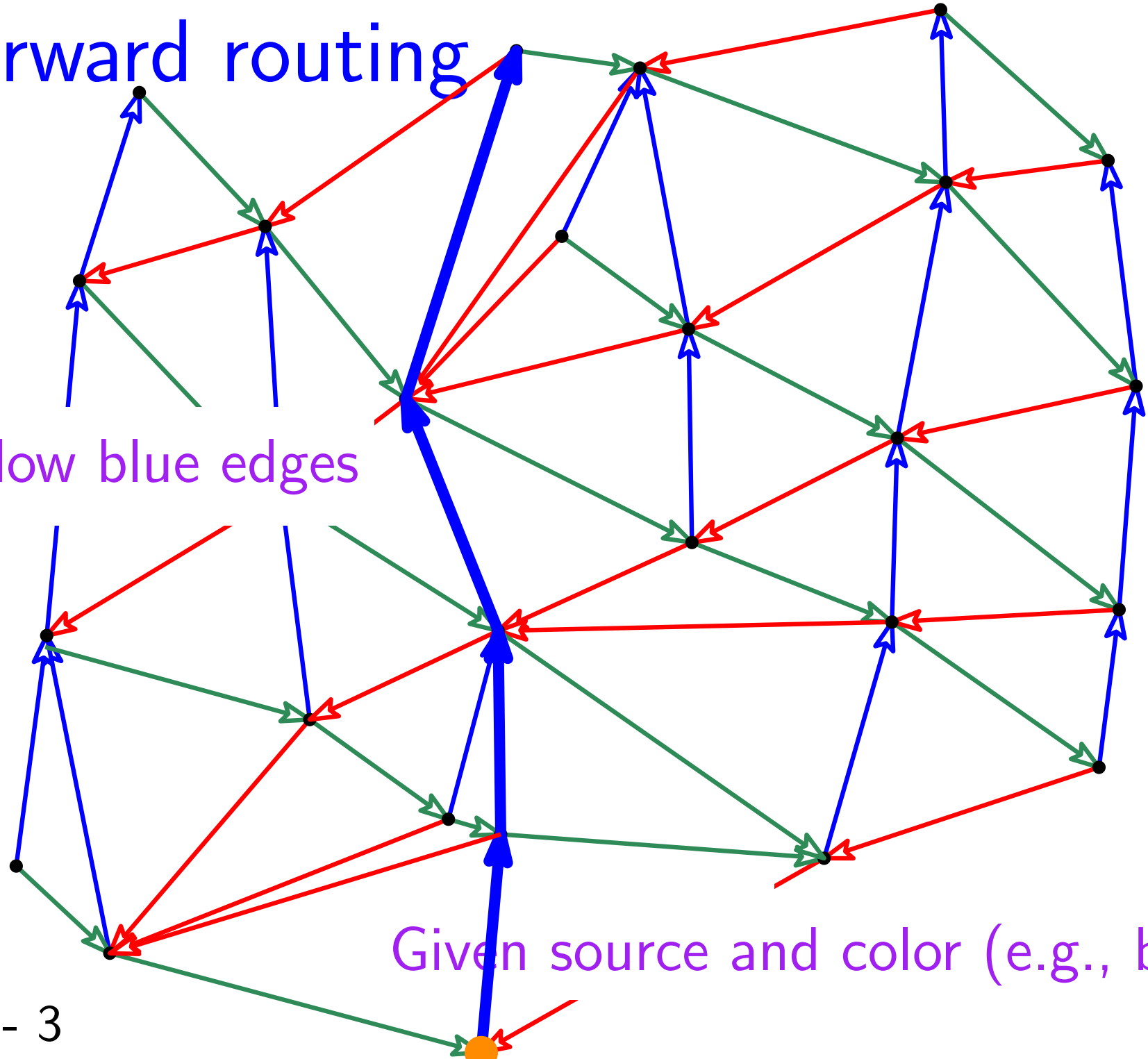


Forward routing



Given source and color (e.g., blue)

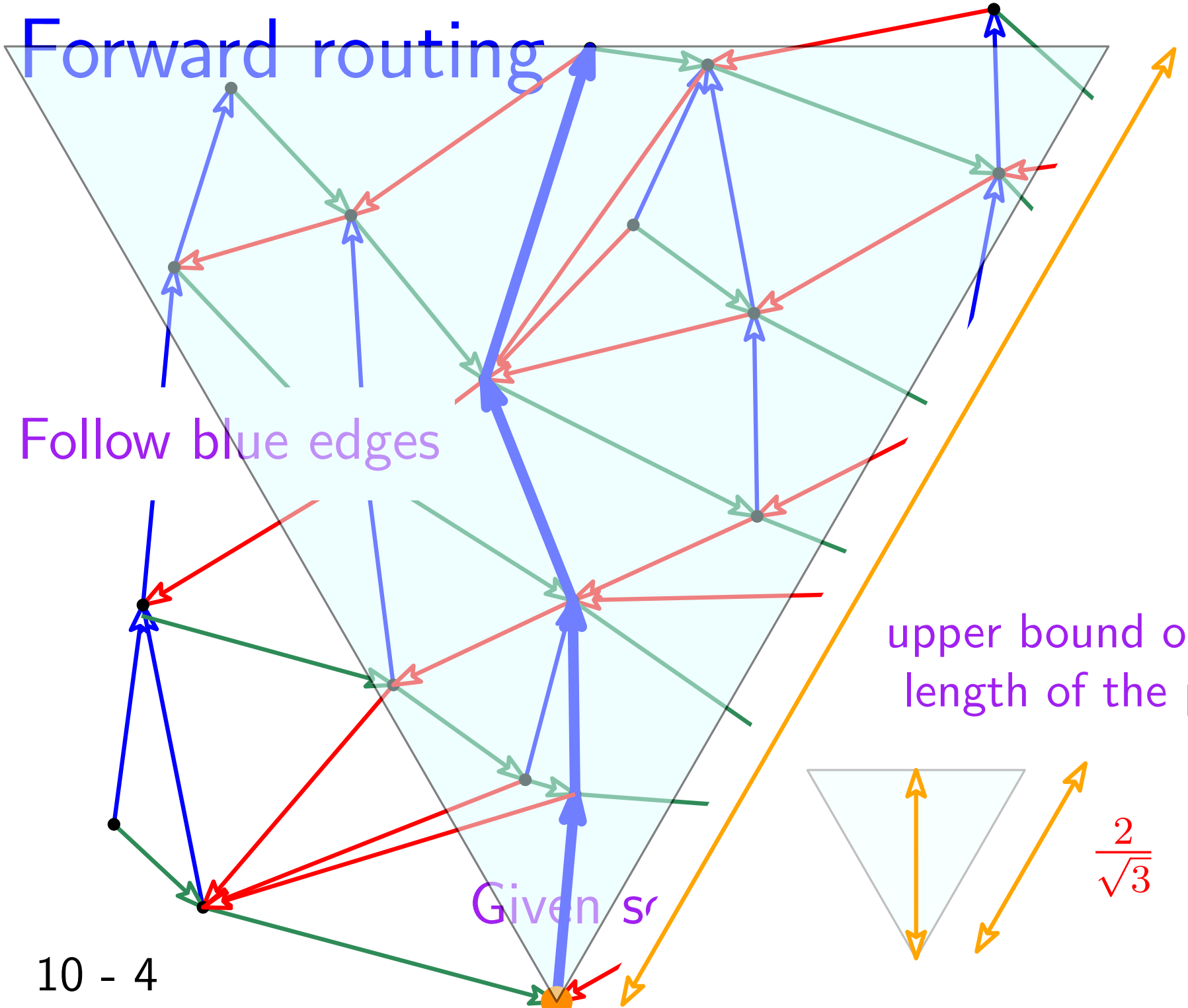
Forward routing



Follow blue edges

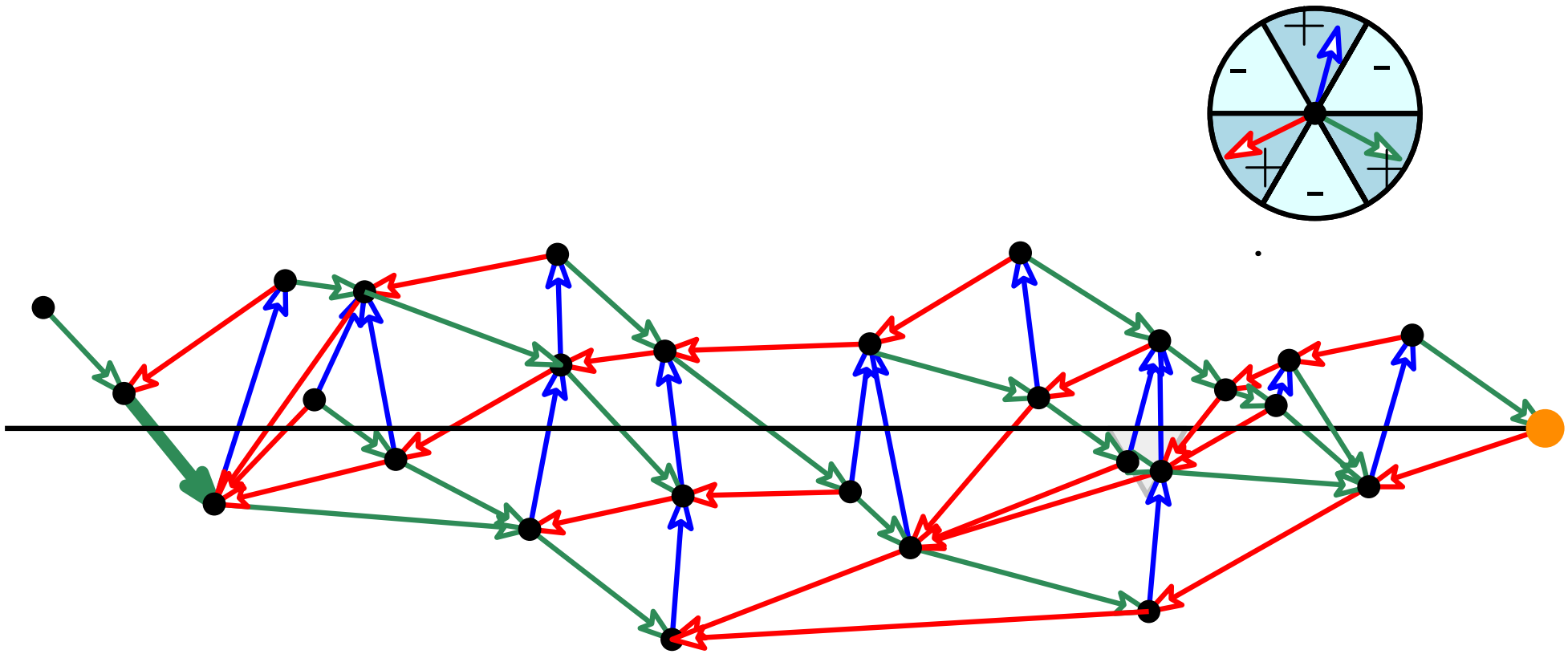
Given source and color (e.g., blue)

Forward routing



Side routing

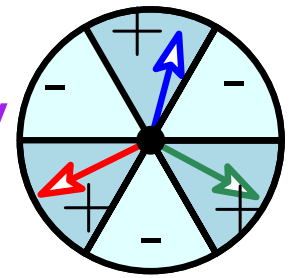
Given target, direction, edge crossing from - to +

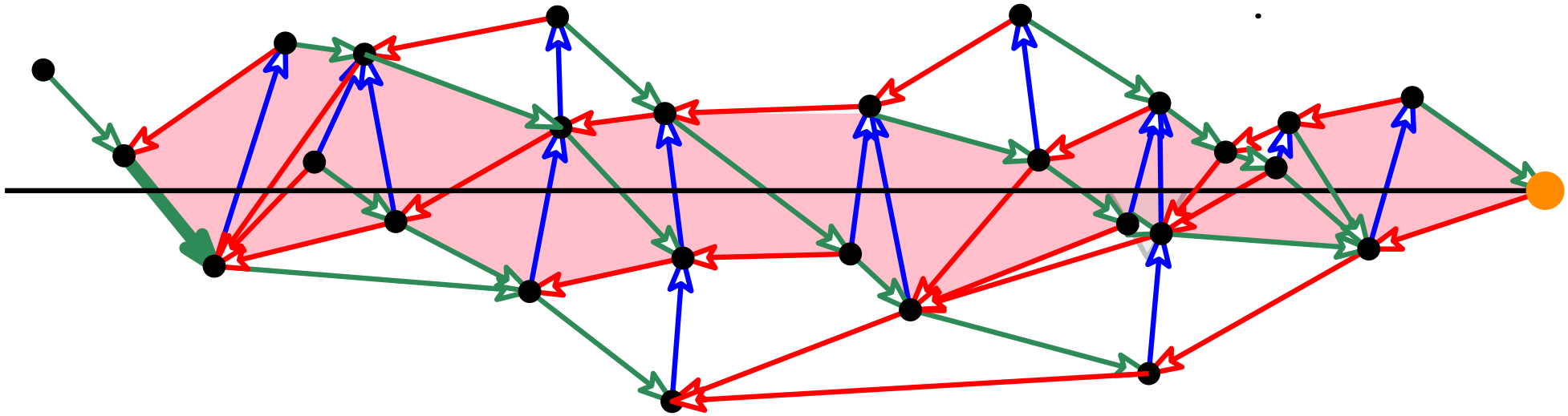


Side routing

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray

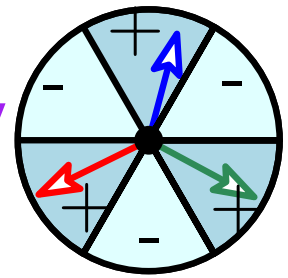


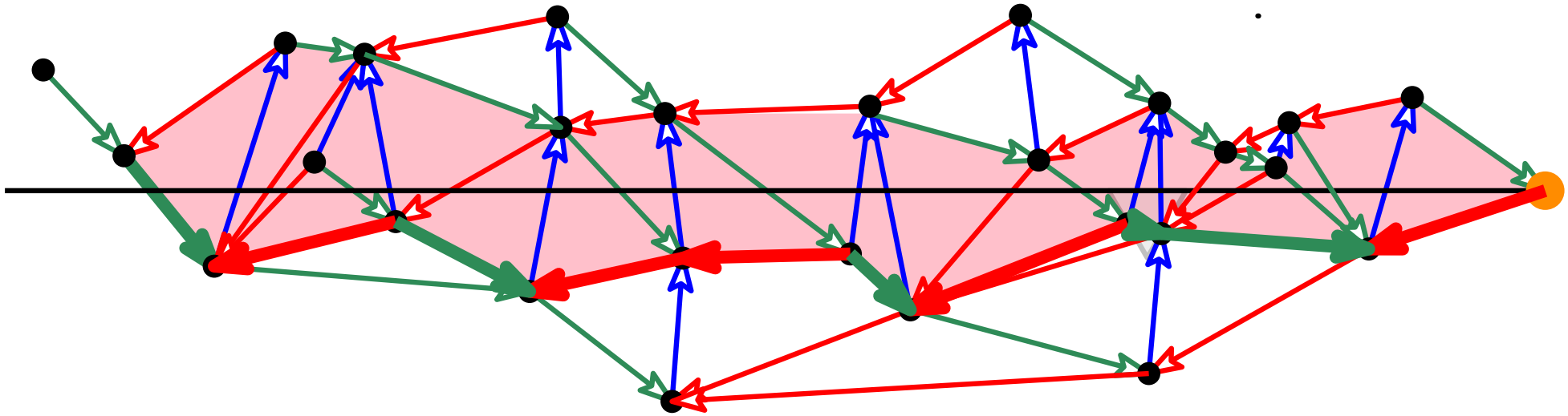


Side routing

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray

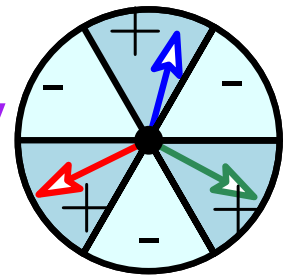


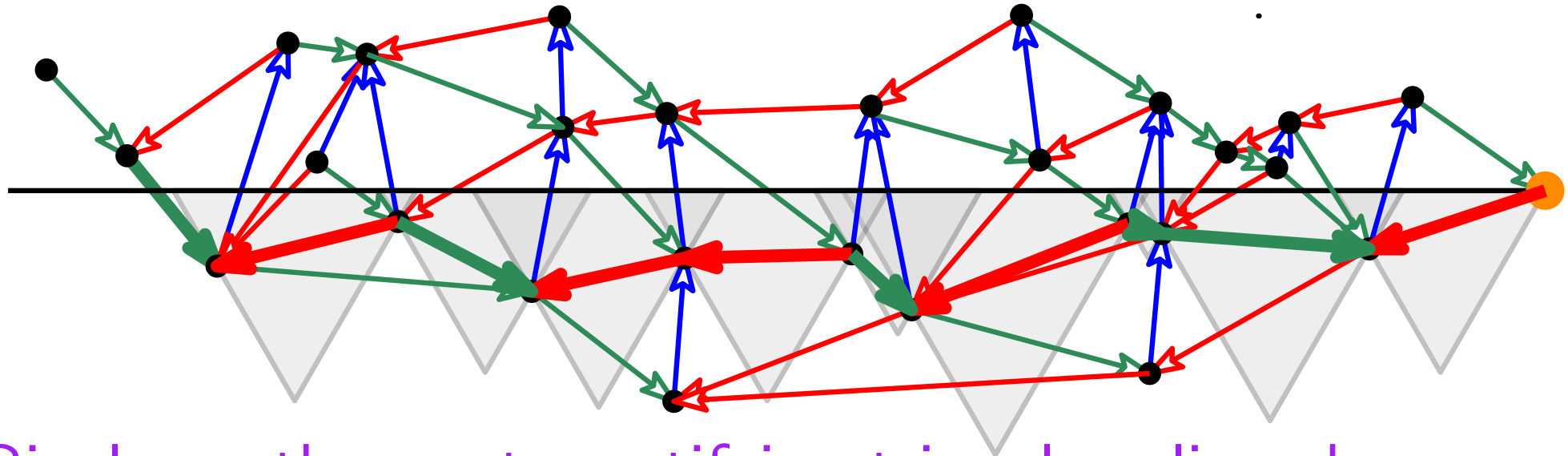


Side routing

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray



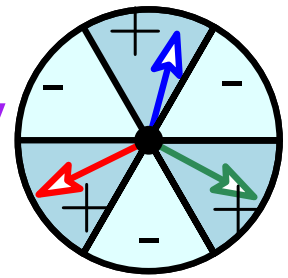


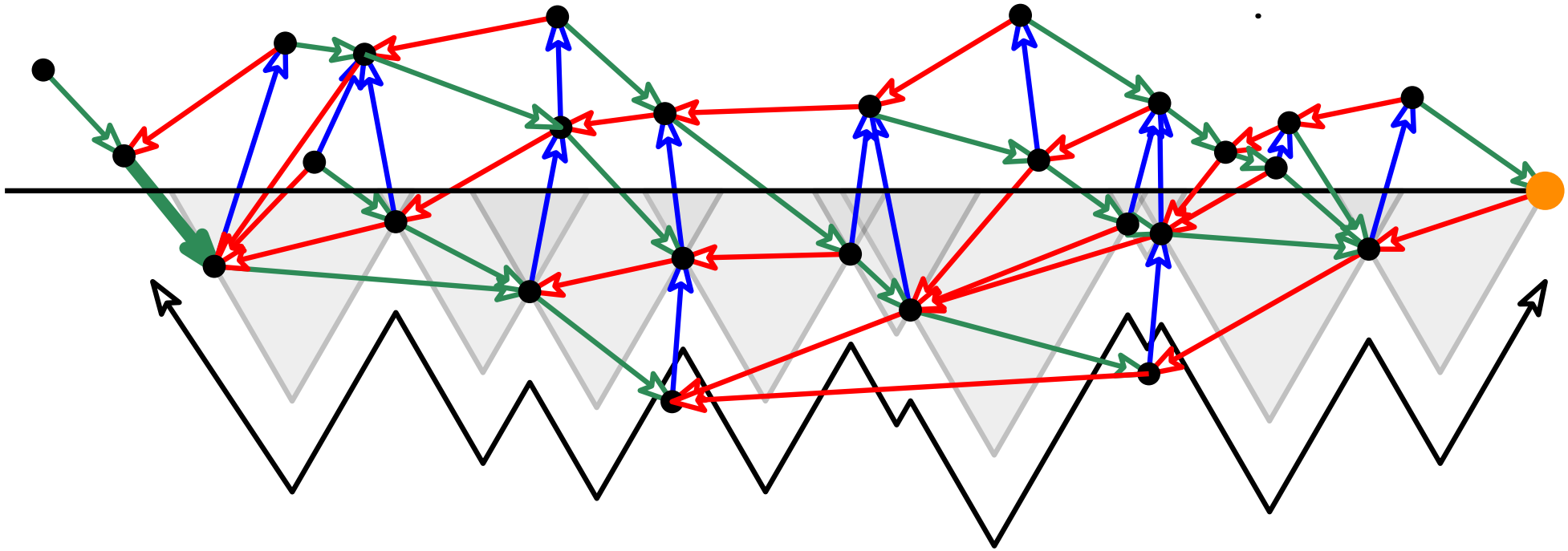
Bicolor path, empty certifying triangles aligned on ray

Side routing

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray



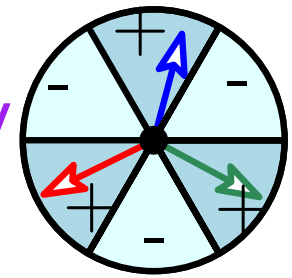


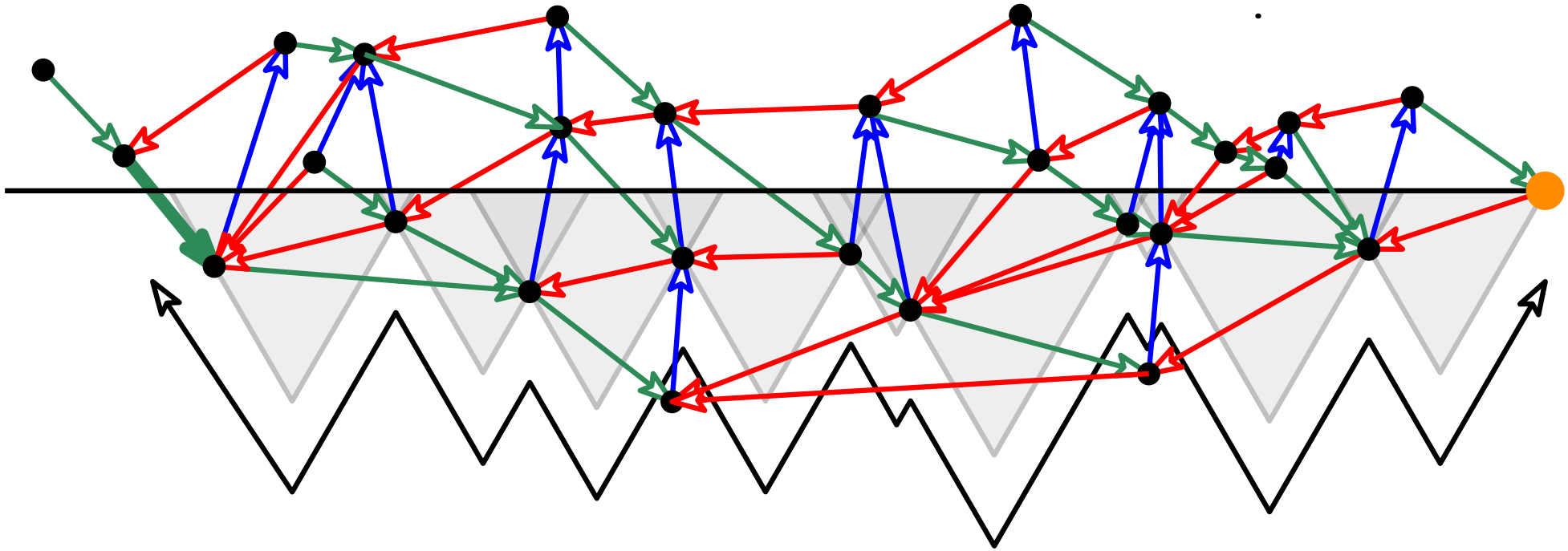
upper bound on the length of the path

Side routing

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray





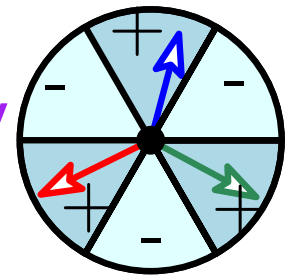
upper bound on the length of the path ≤ 2

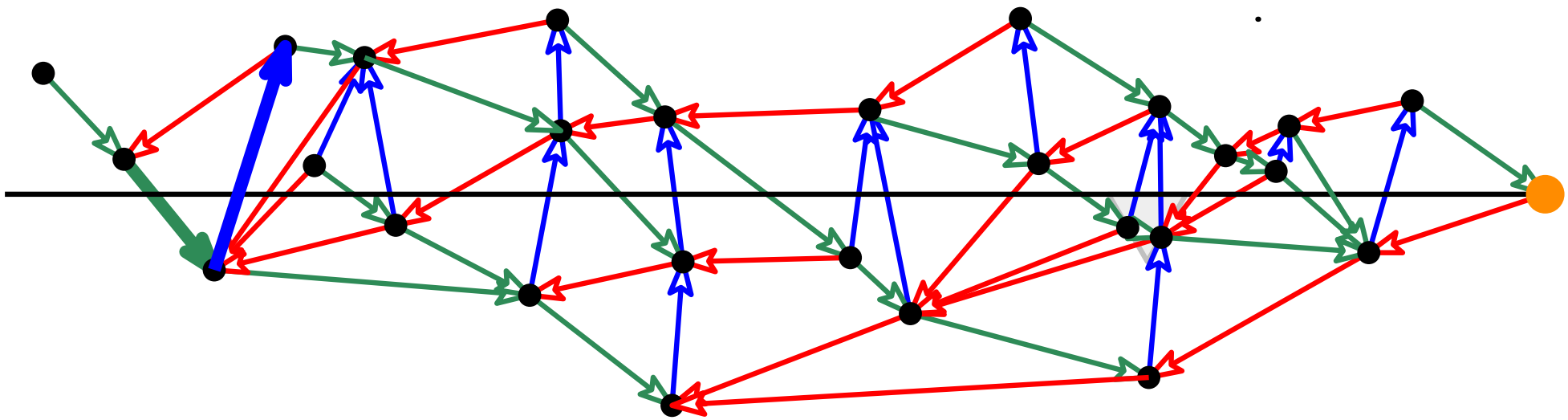
Side routing

Does not work from + to -

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray



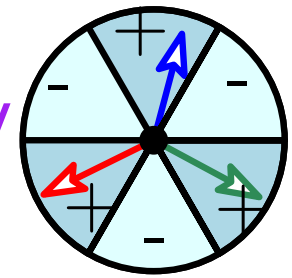


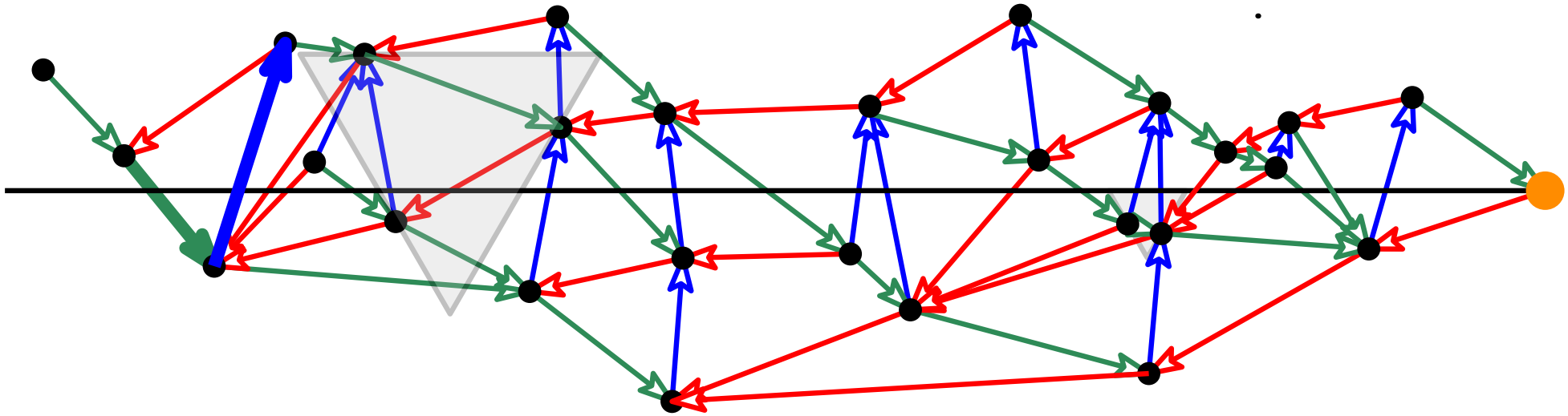
Side routing

Does not work from + to -

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray





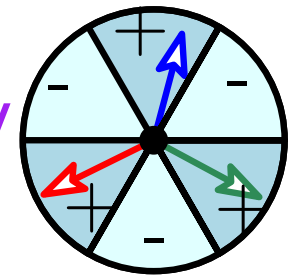
cannot use circumtriangle to bound length

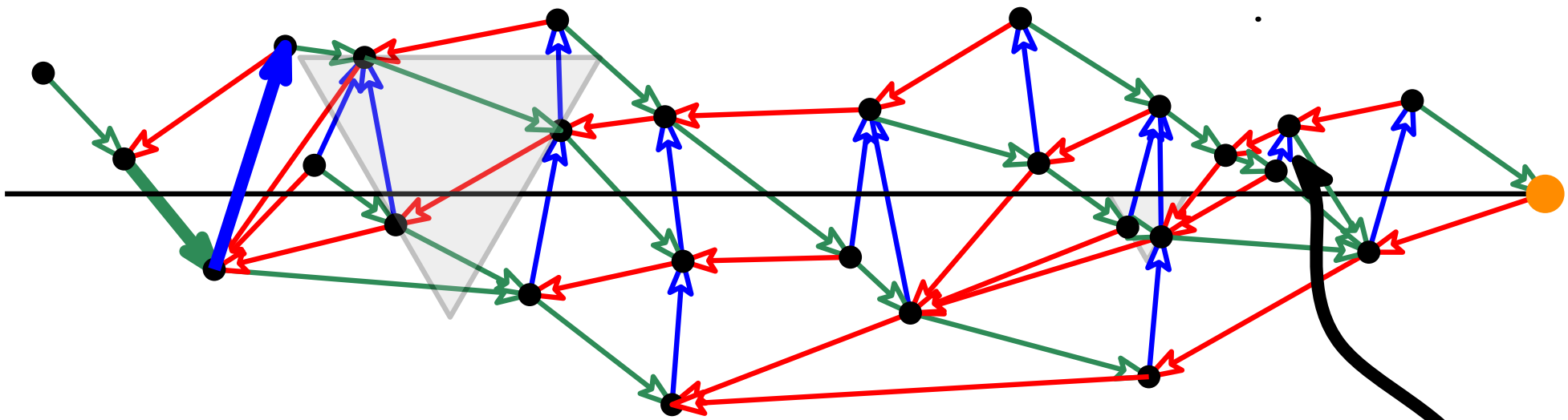
Side routing

Does not work from + to -

Given target, direction, edge crossing from - to +

Use boundary of triangles crossing the ray

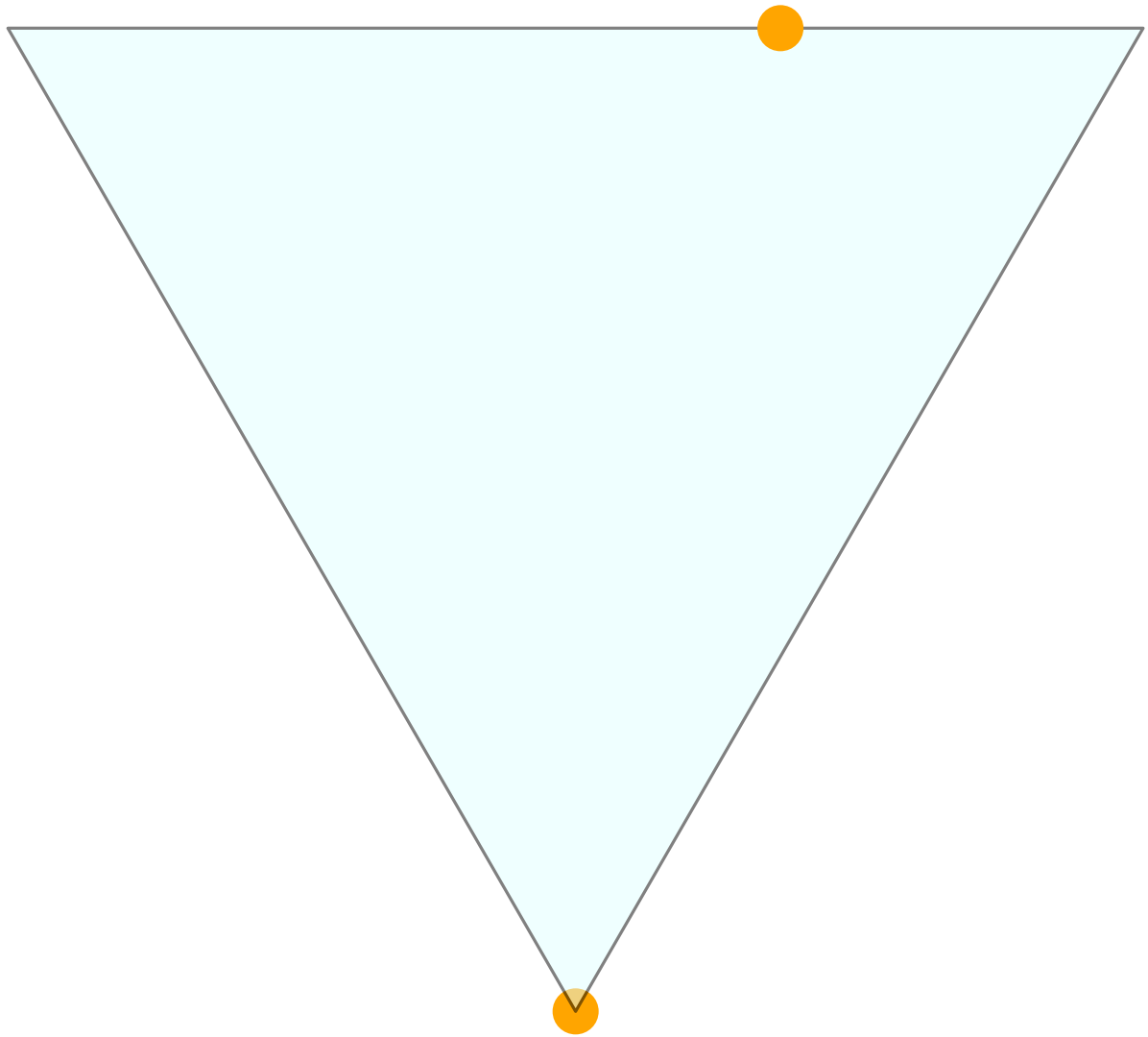




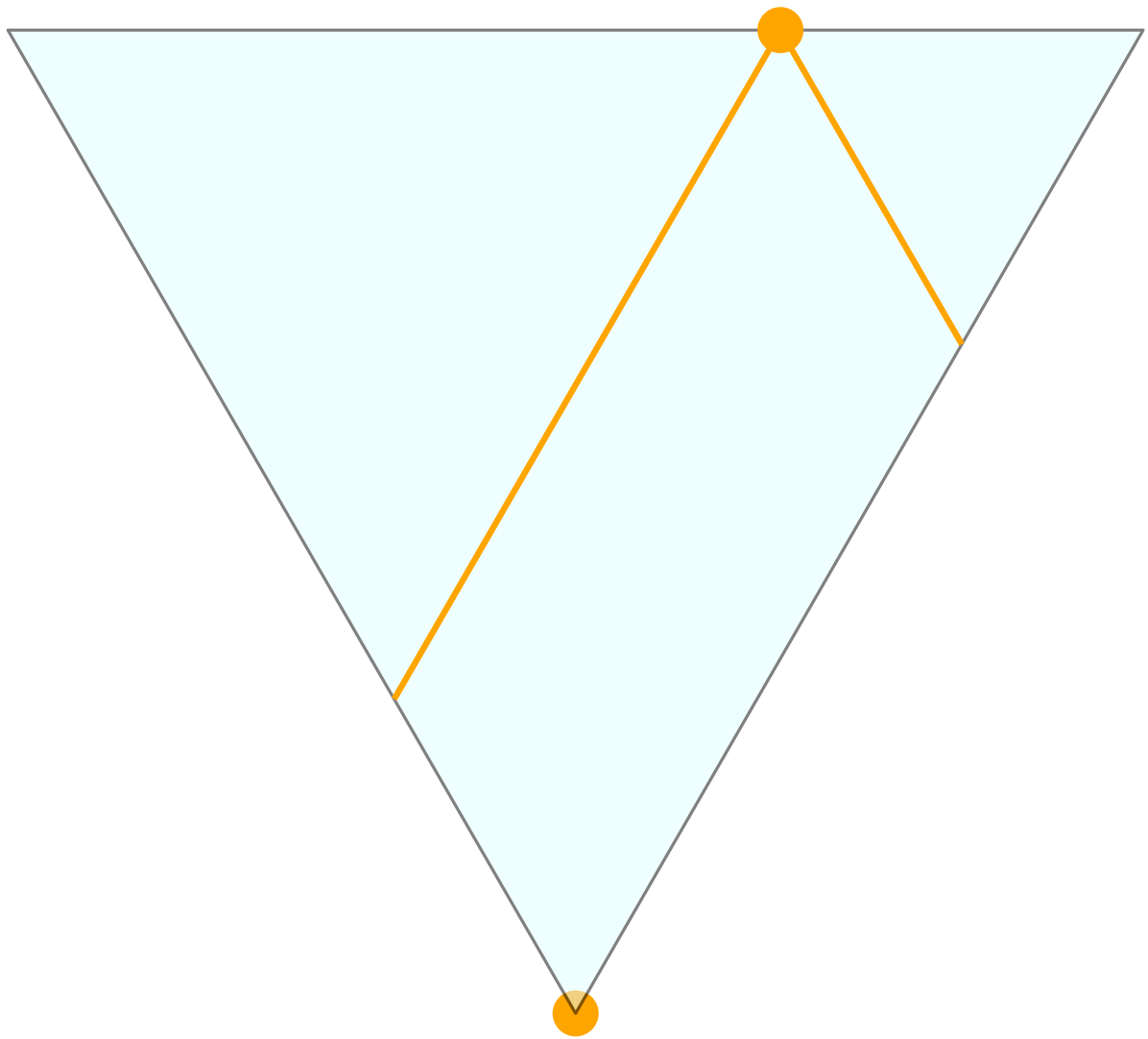
cannot use circumtriangle to bound length

path is not bicolor

Positive routing

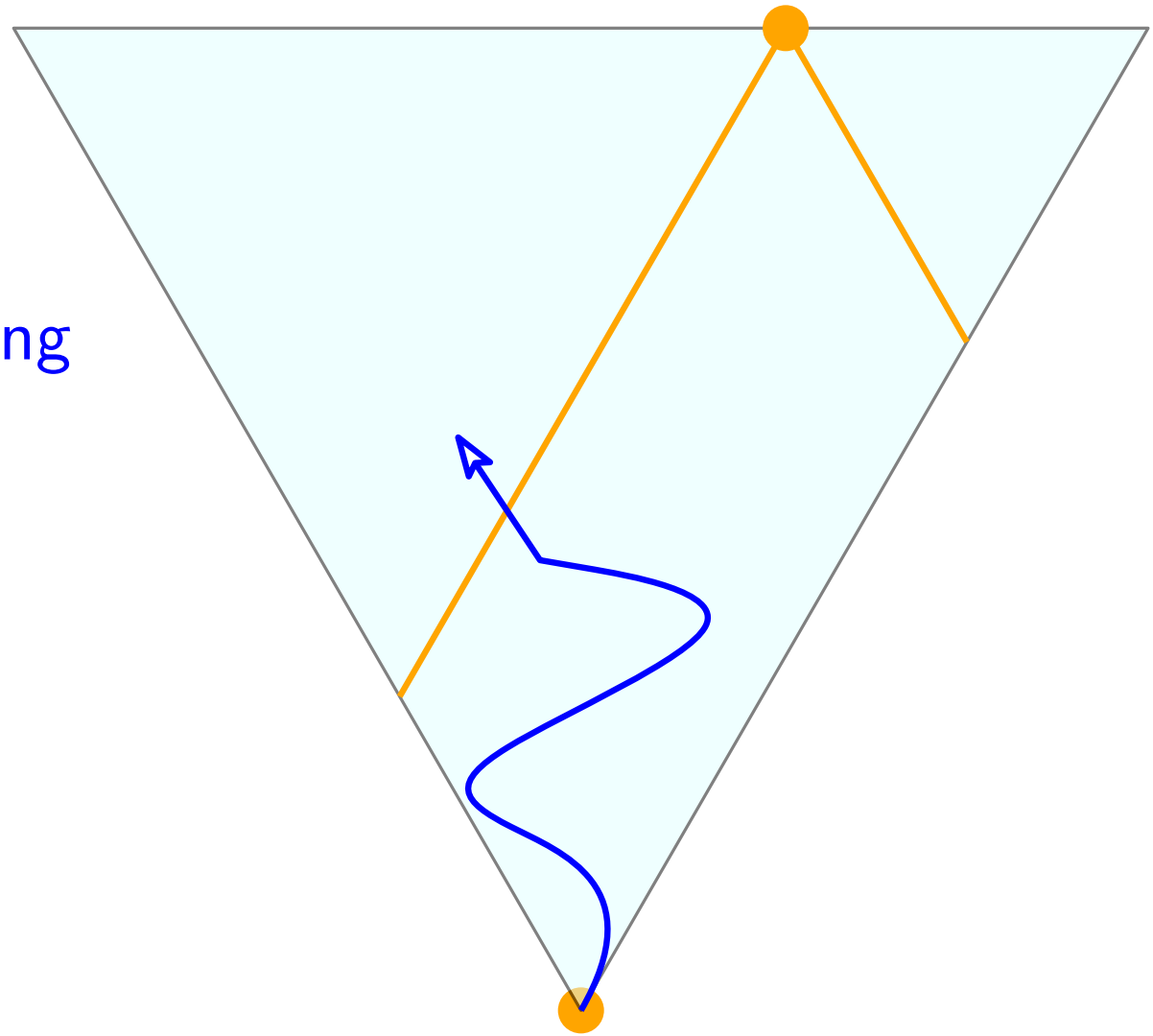


Positive routing



Positive routing

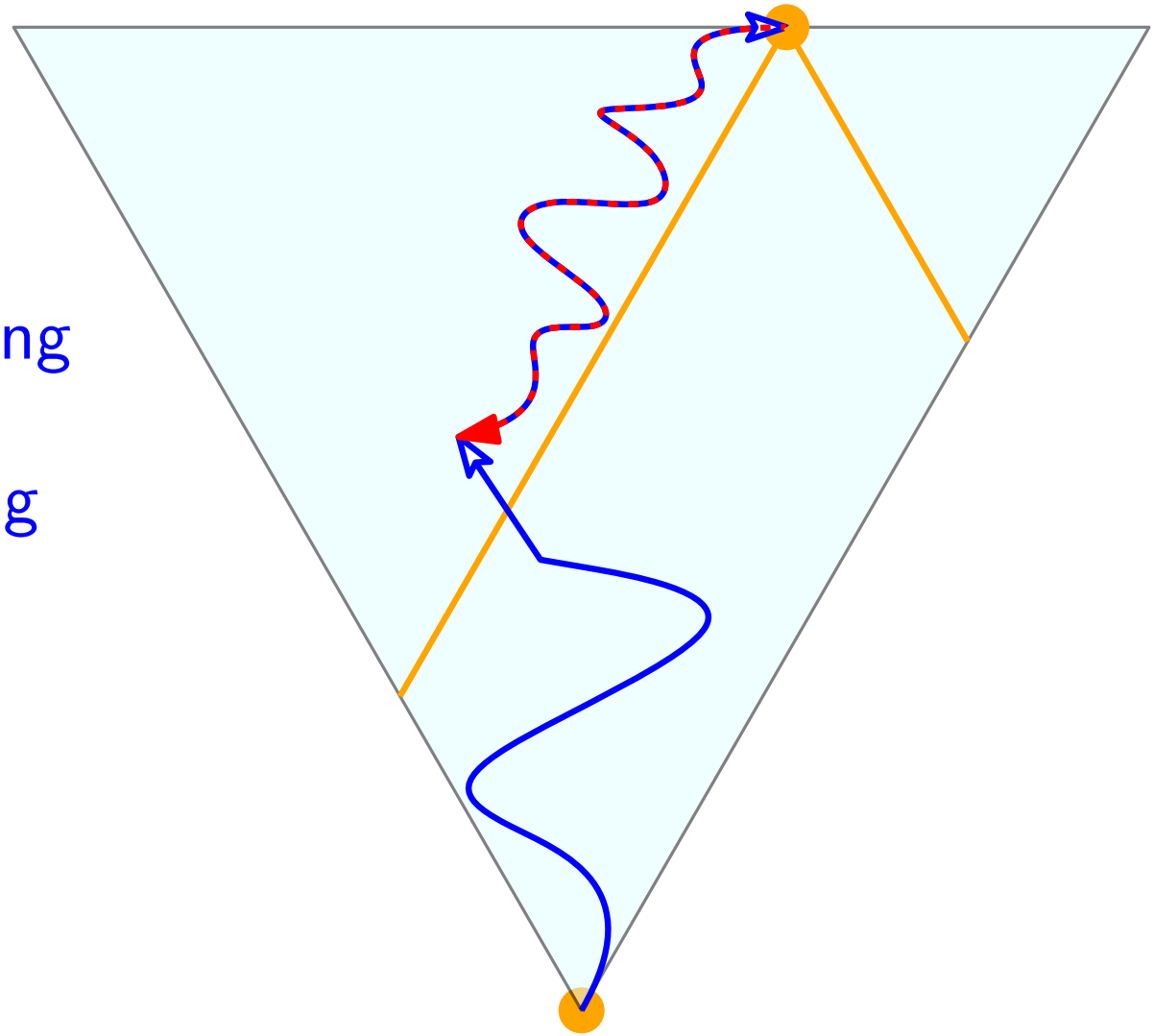
Forward routing



Positive routing

Forward routing

\+ Side routing

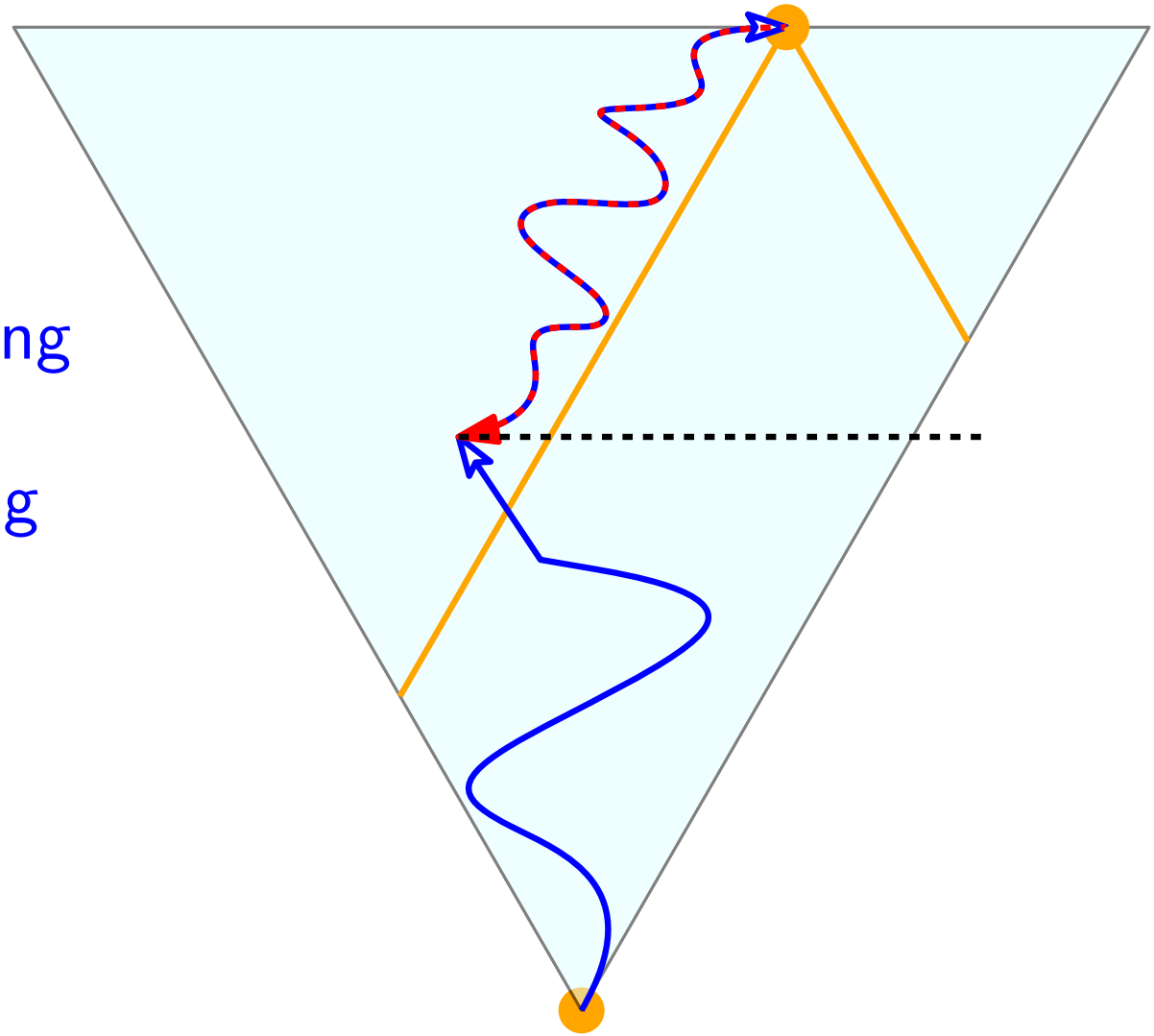


Positive routing

Forward routing

\+ Side routing

Length=

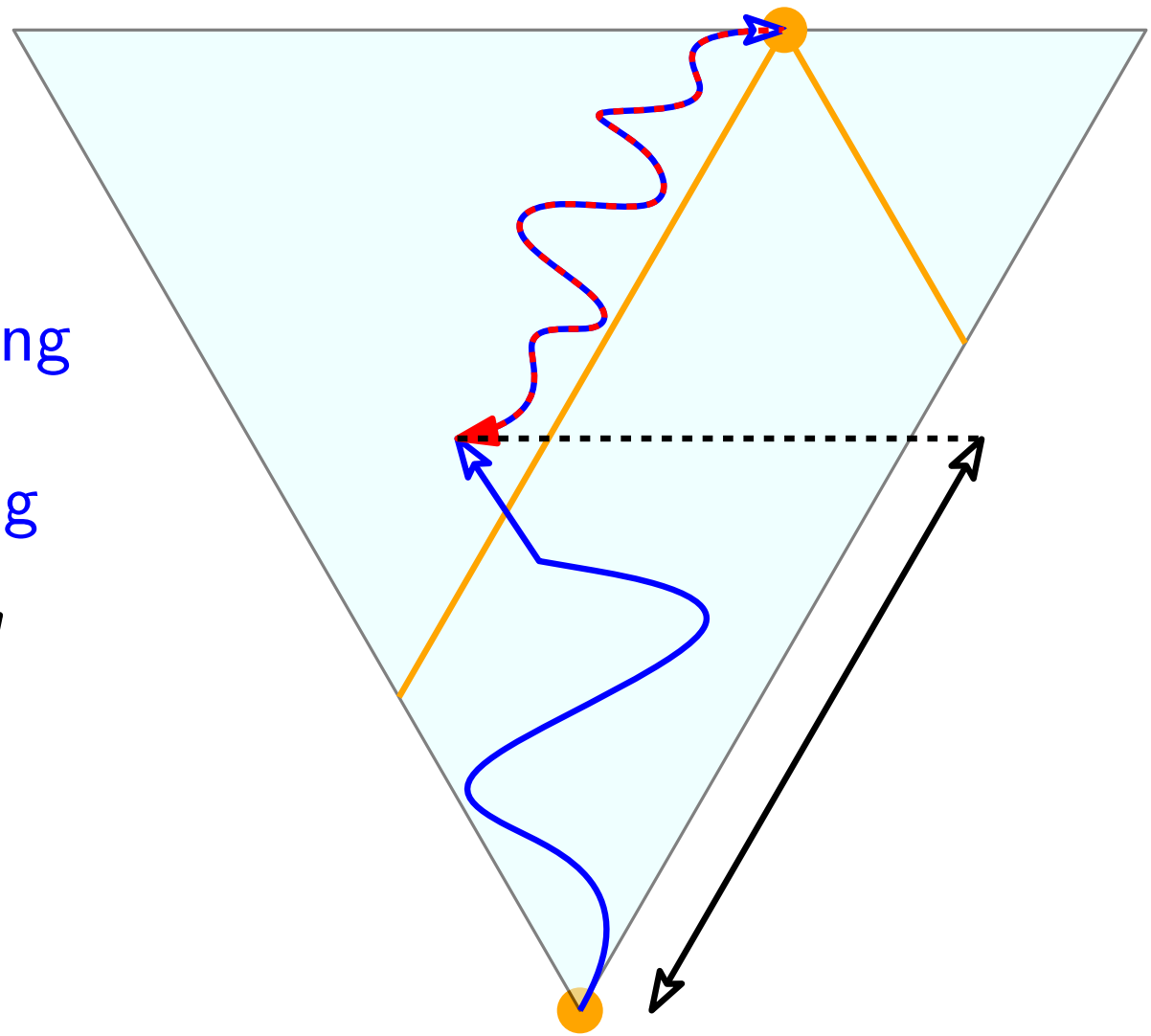


Positive routing

Forward routing

+ Side routing

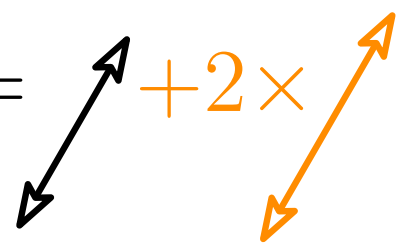
Length=



Positive routing

Forward routing

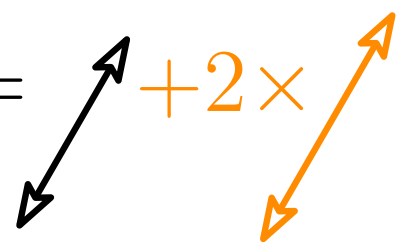
$+$ Side routing

Length$=$  $+2\times$

Positive routing

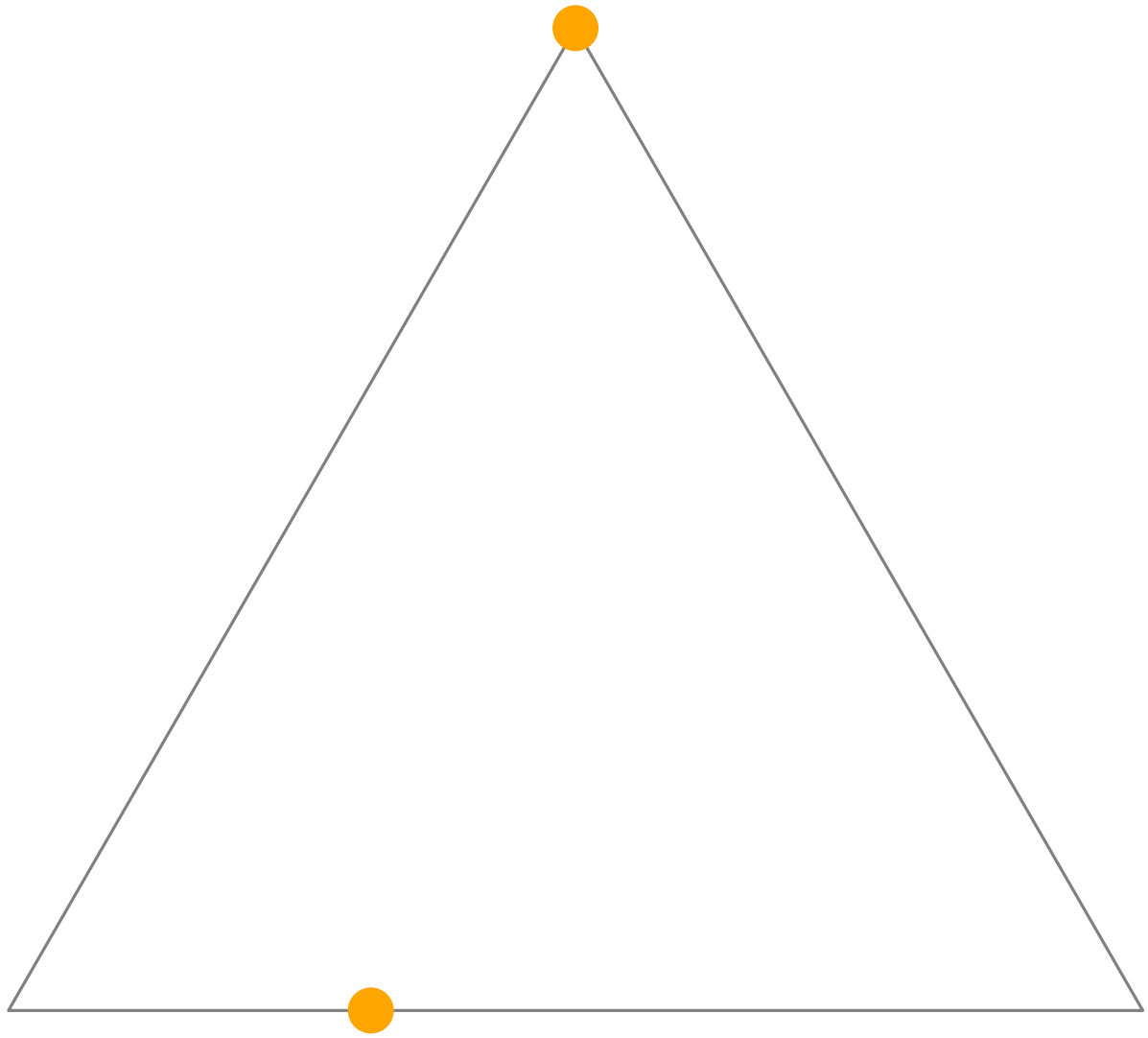
Forward routing

$+$ Side routing

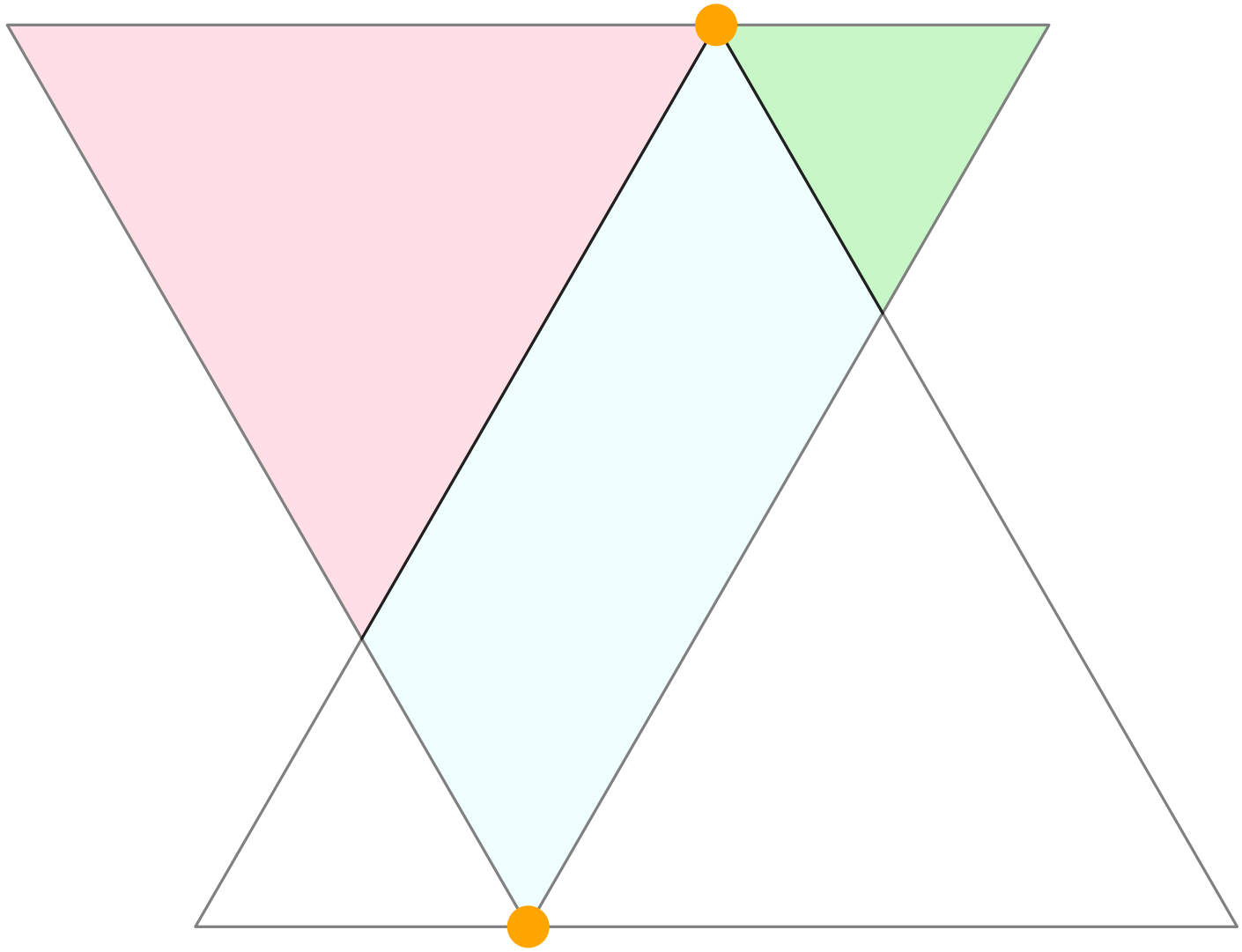
Length$=$  $+2\times$

Stretch ≤ 2

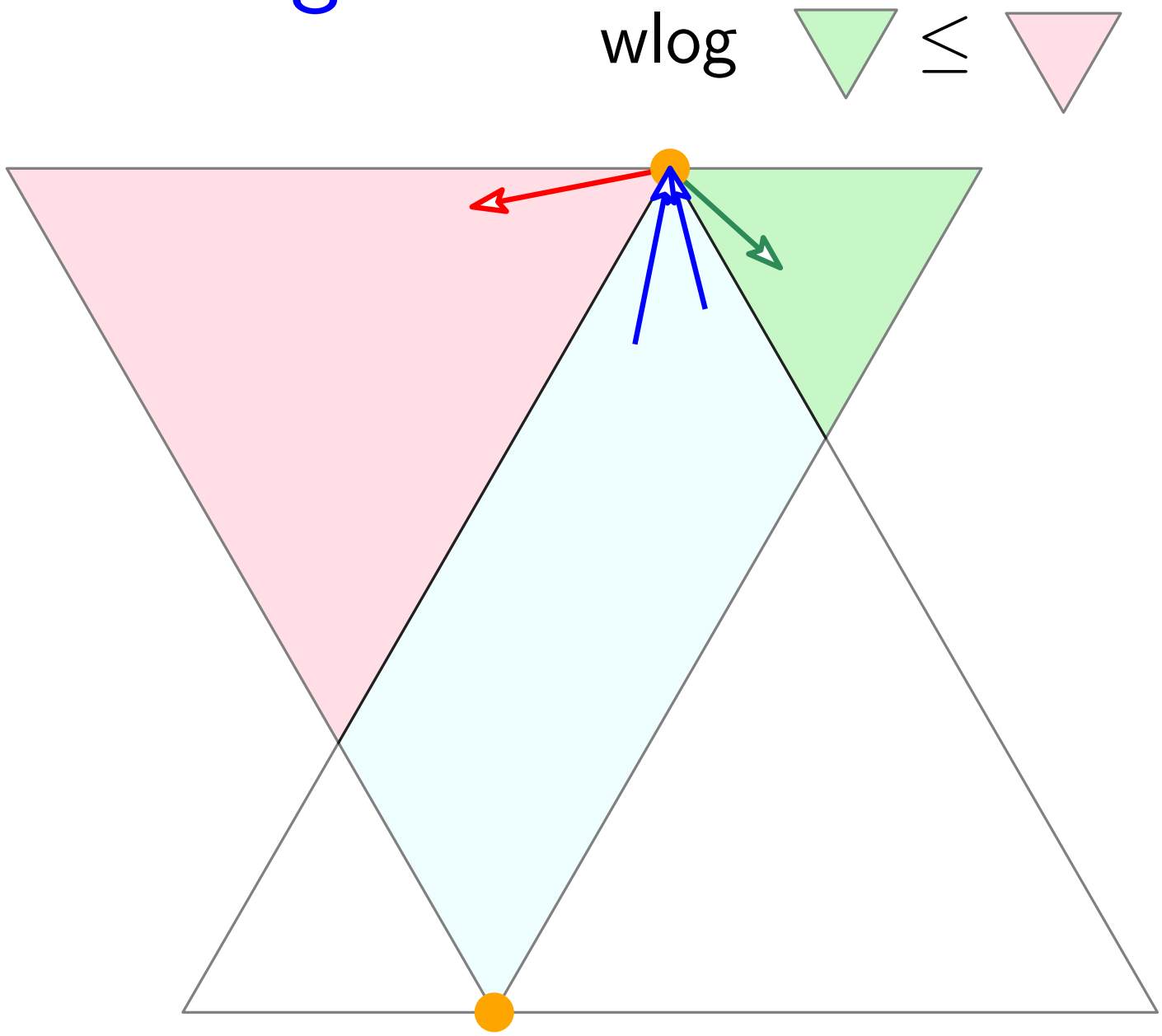
Negative routing



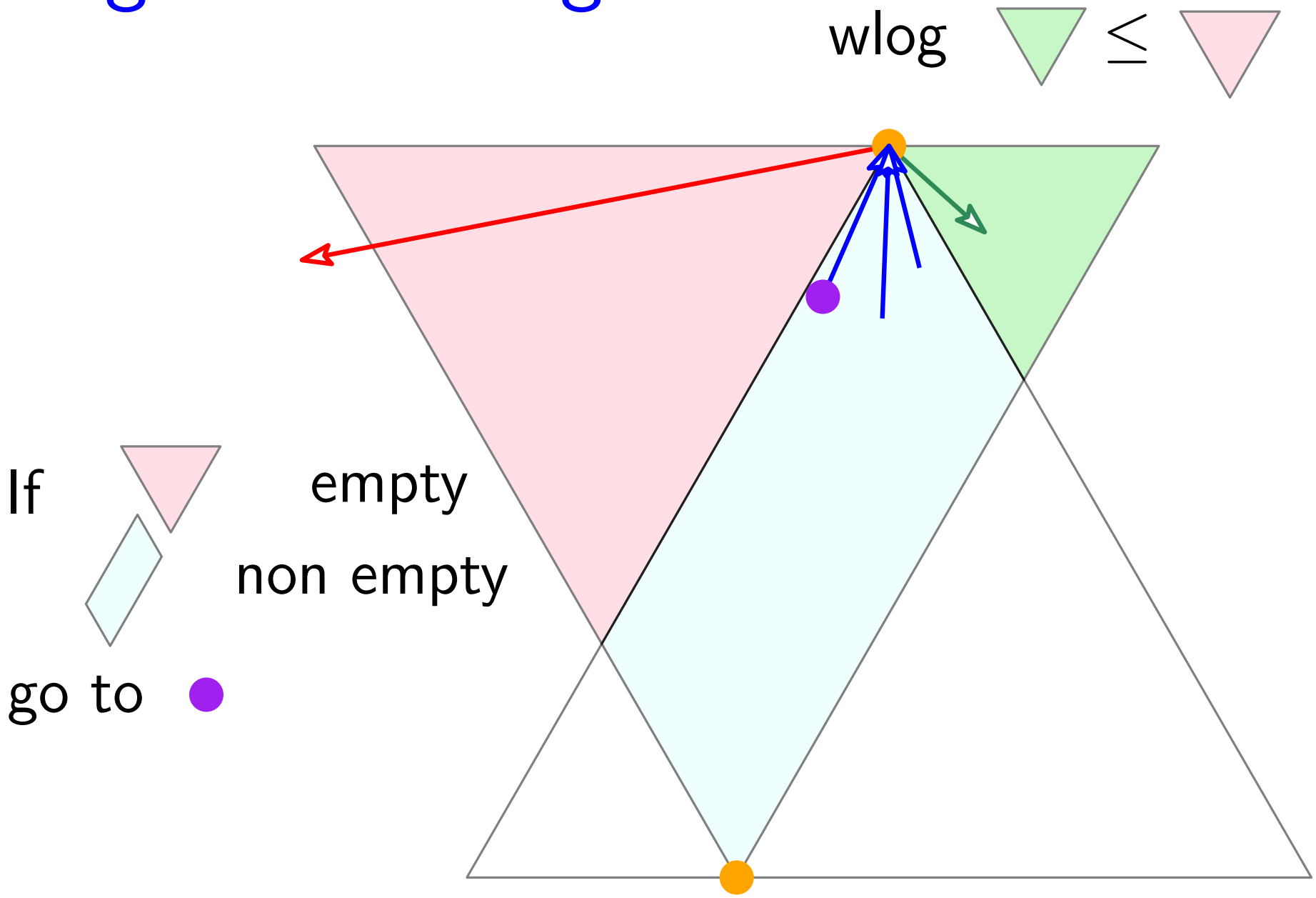
Negative routing



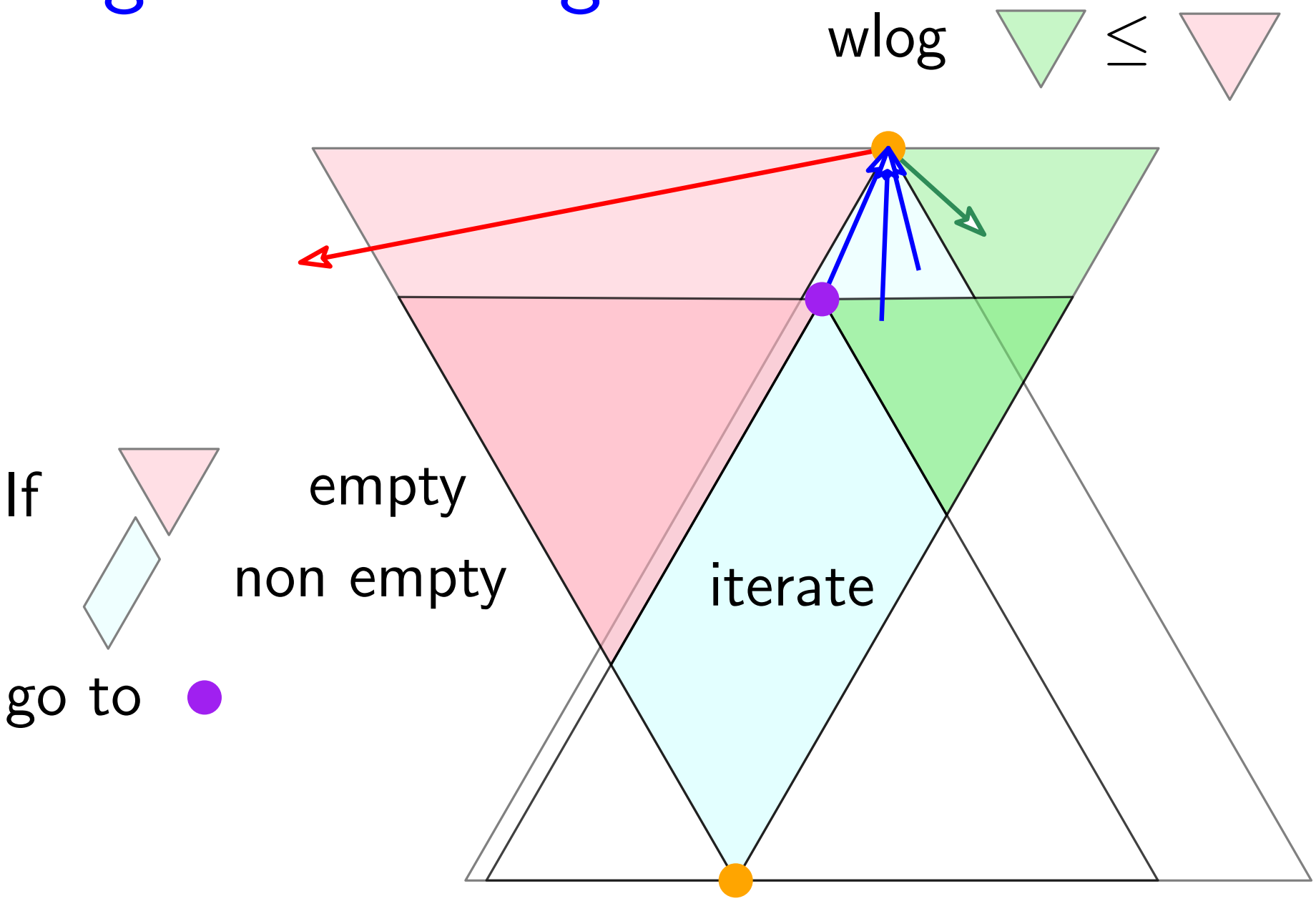
Negative routing



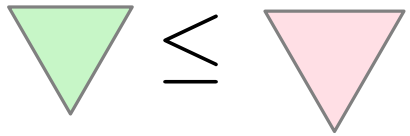
Negative routing

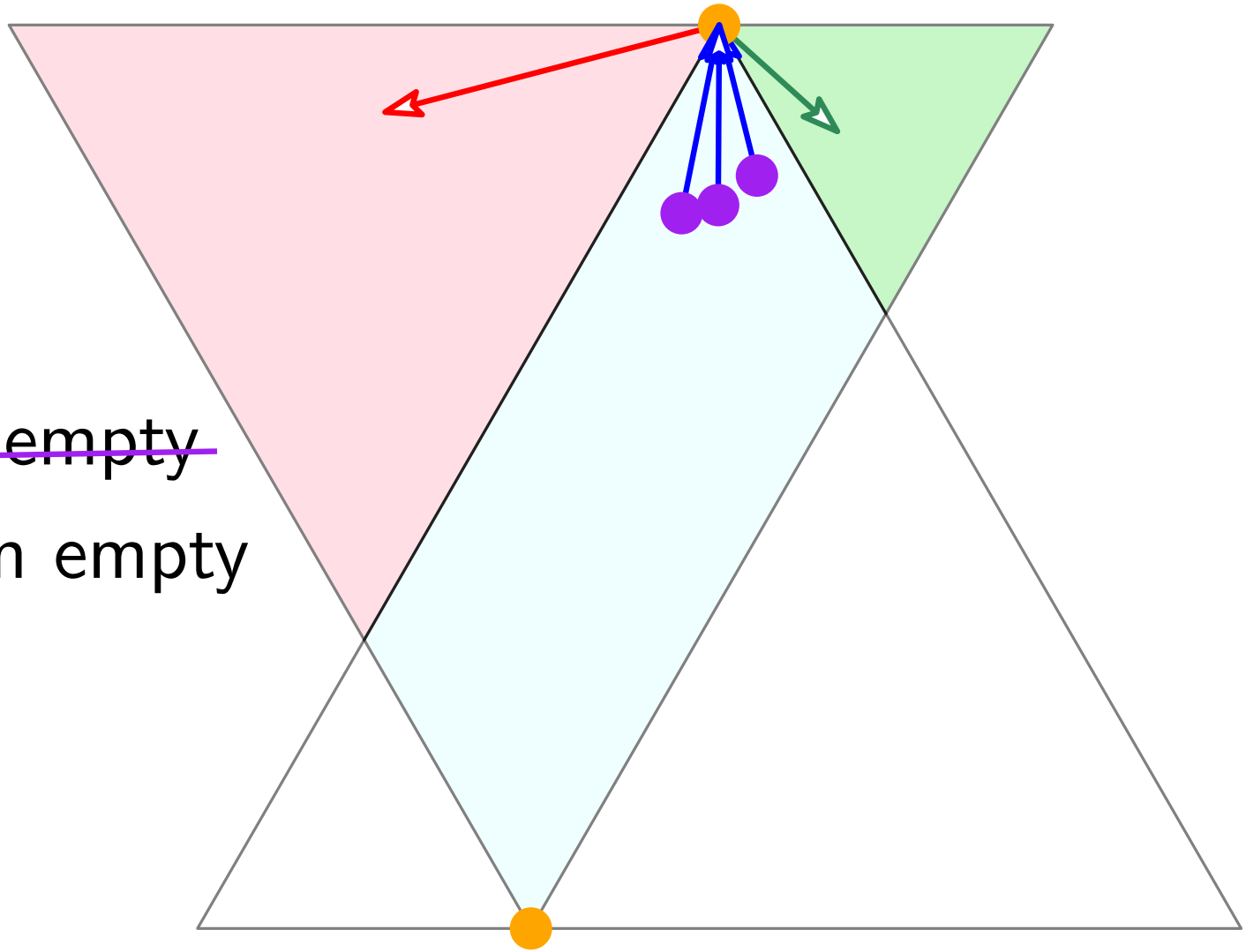


Negative routing

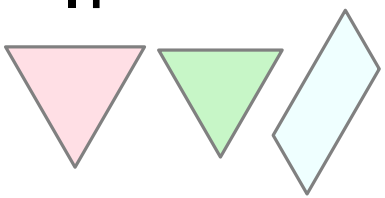


Negative routing

wlog  $\leq$

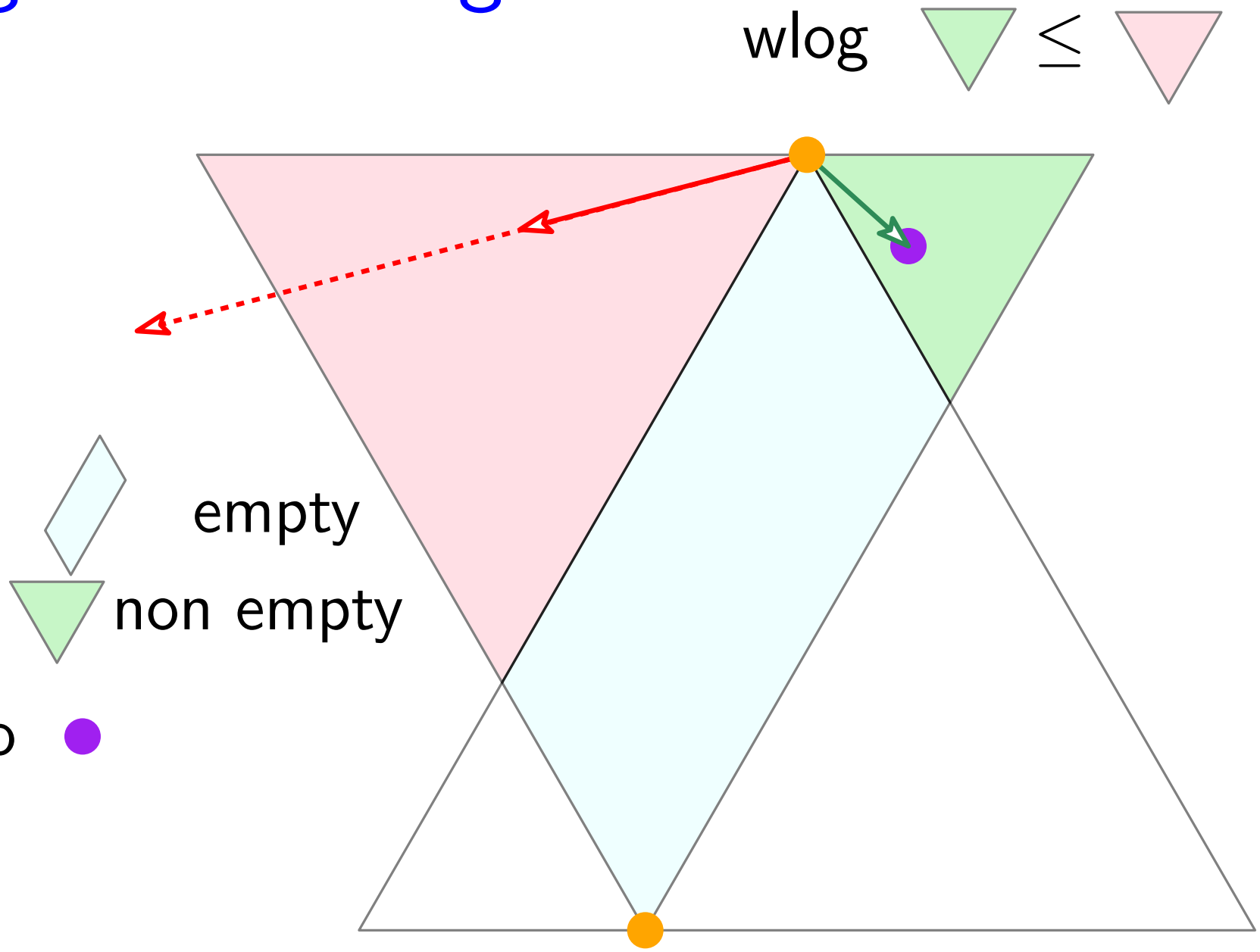


If ~~empty~~

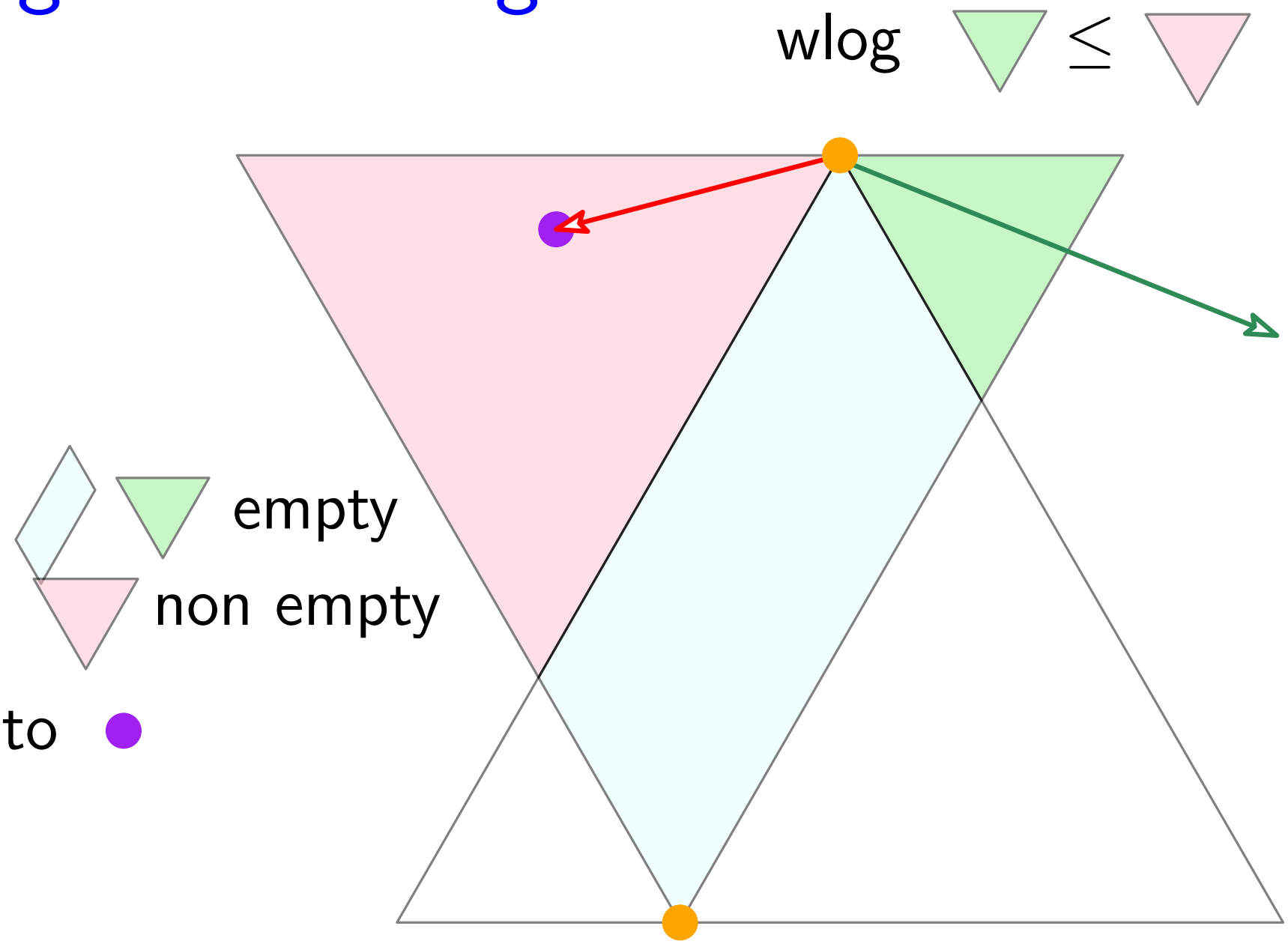
 non empty

go to ●

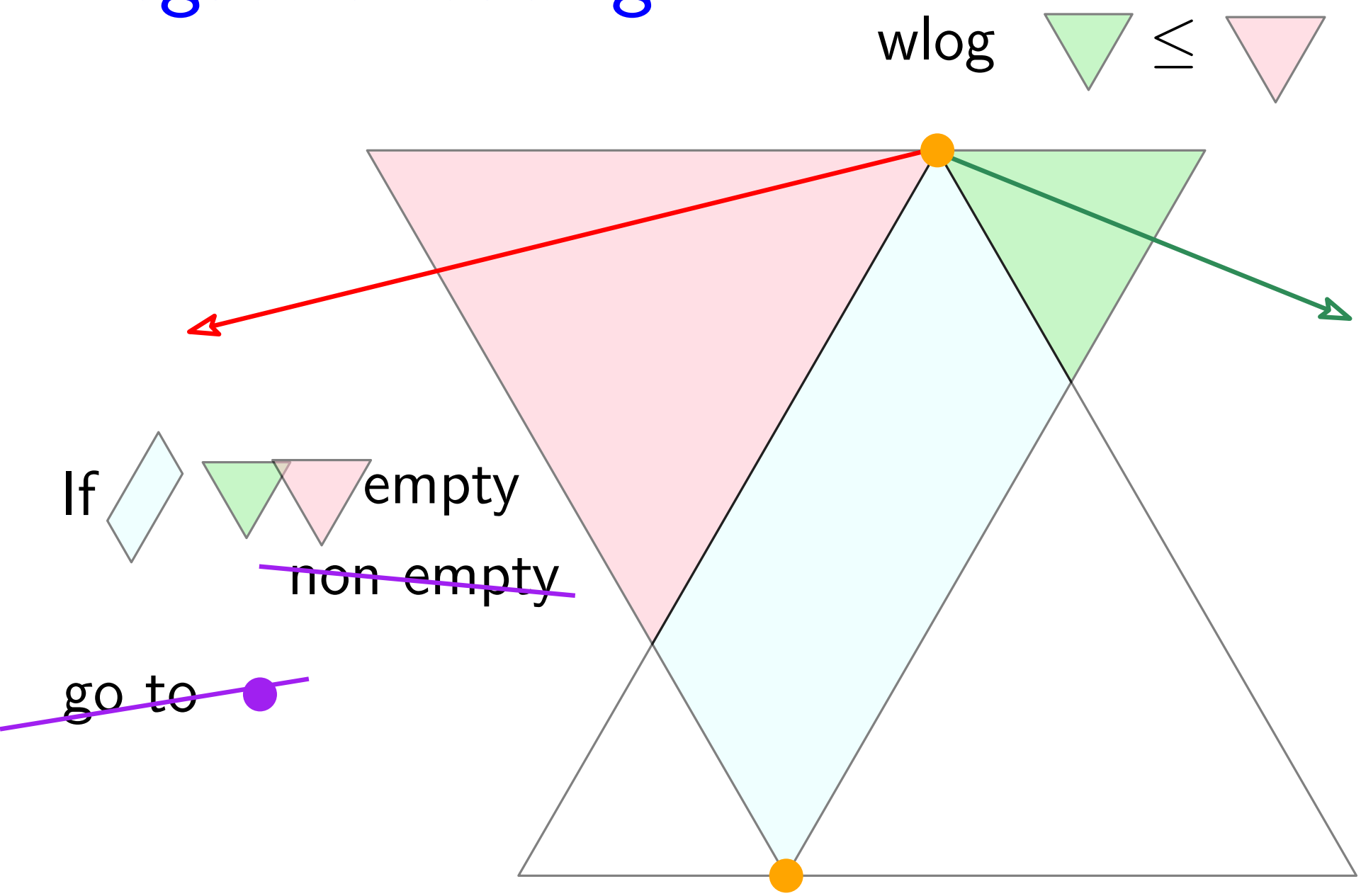
Negative routing



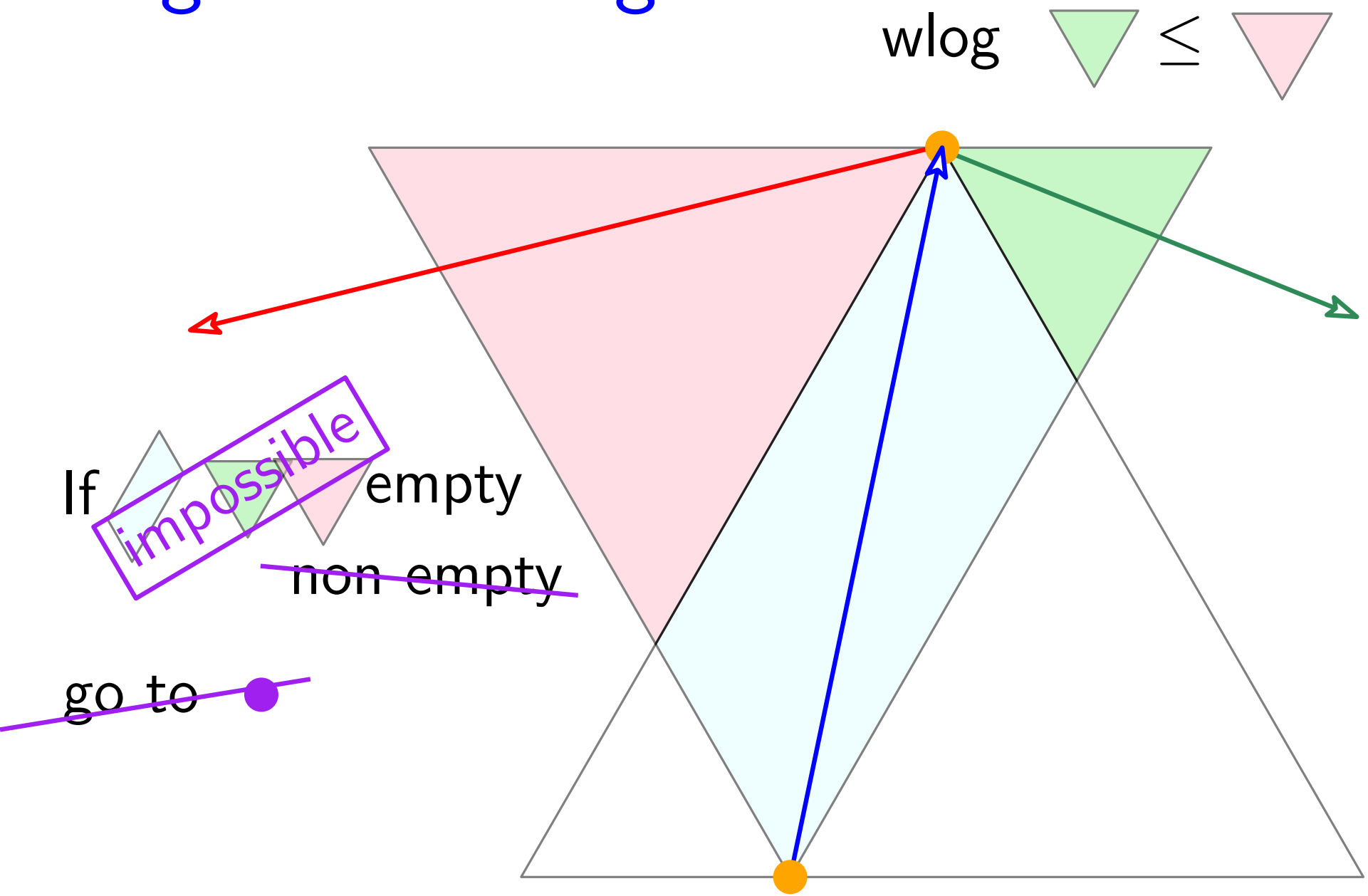
Negative routing



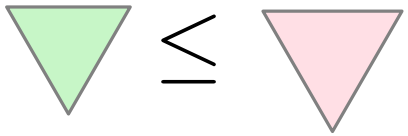
Negative routing

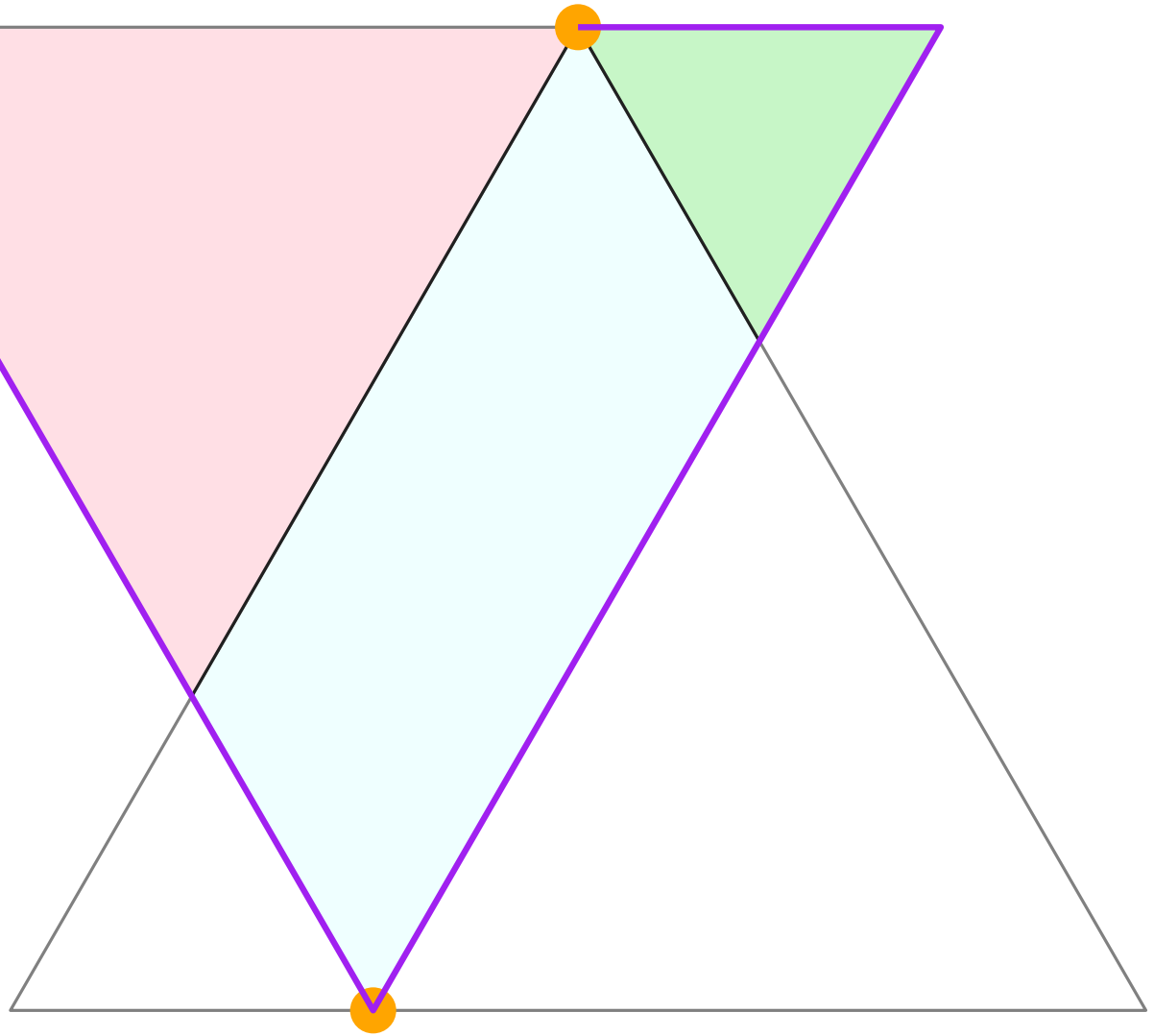


Negative routing



Negative routing

wlog  $\leq$

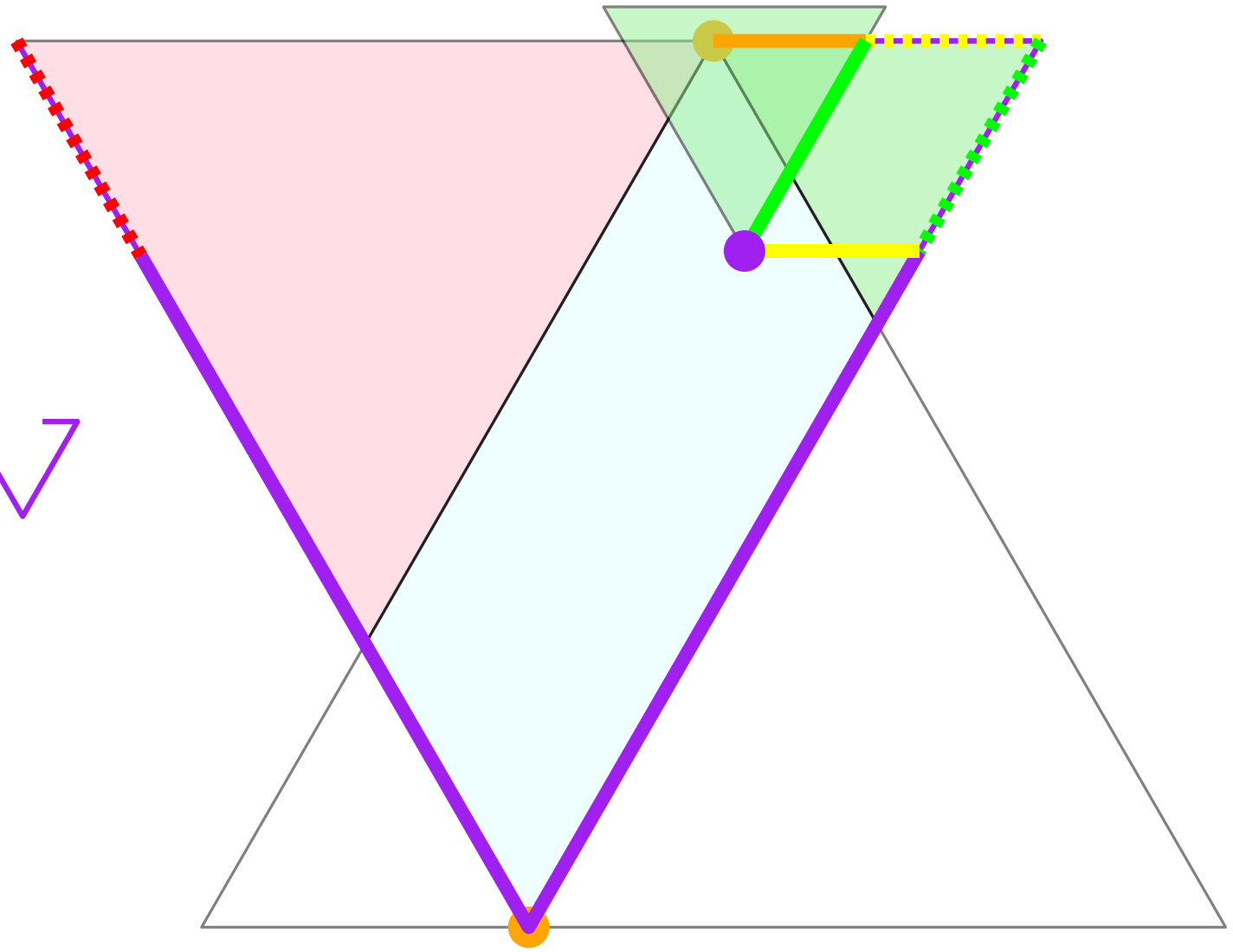


Length $\leq$ 

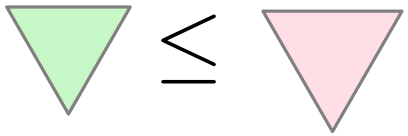
Negative routing

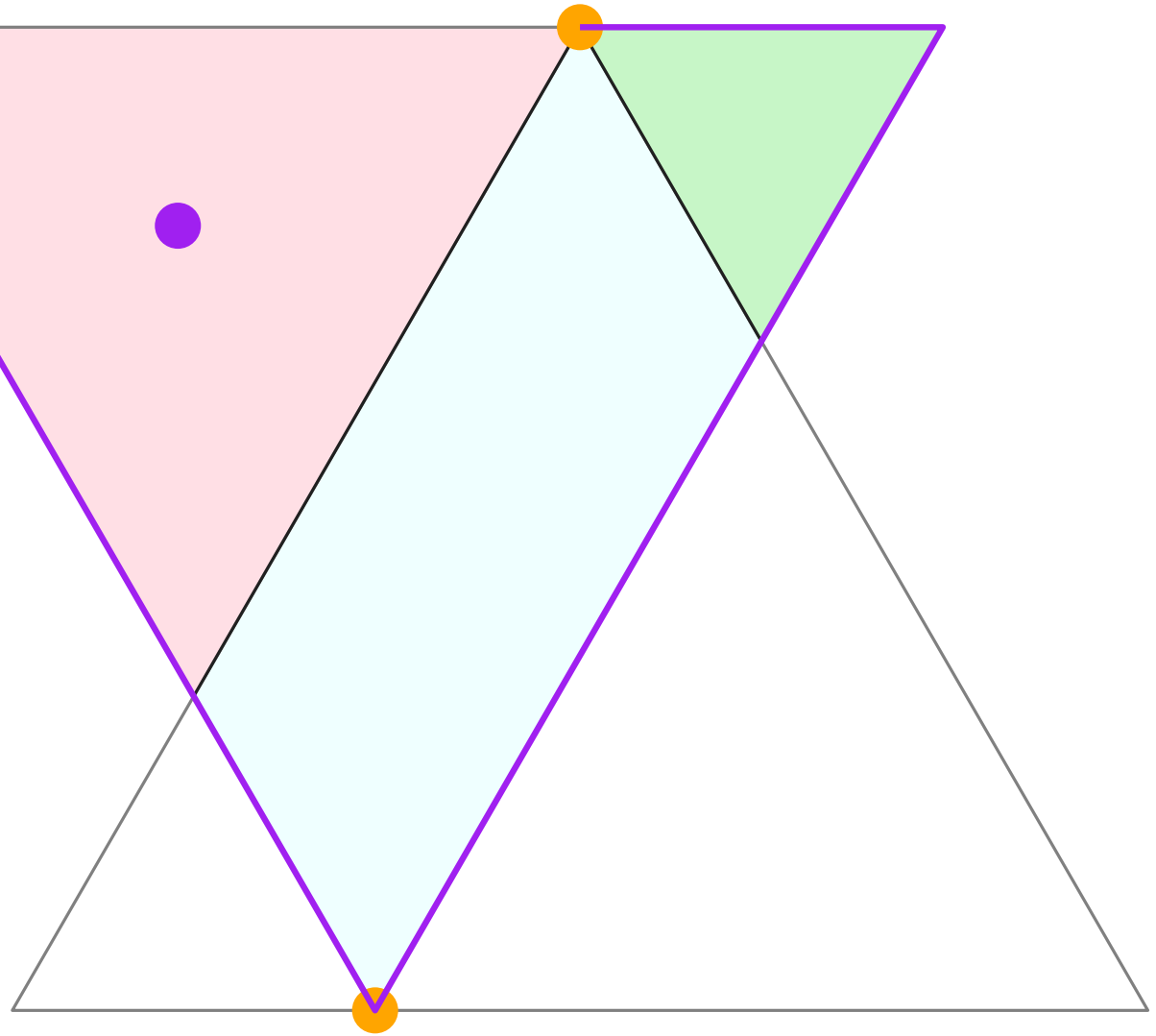
wlog $\leq$

Length $\leq$



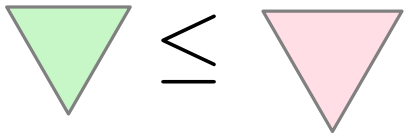
Negative routing

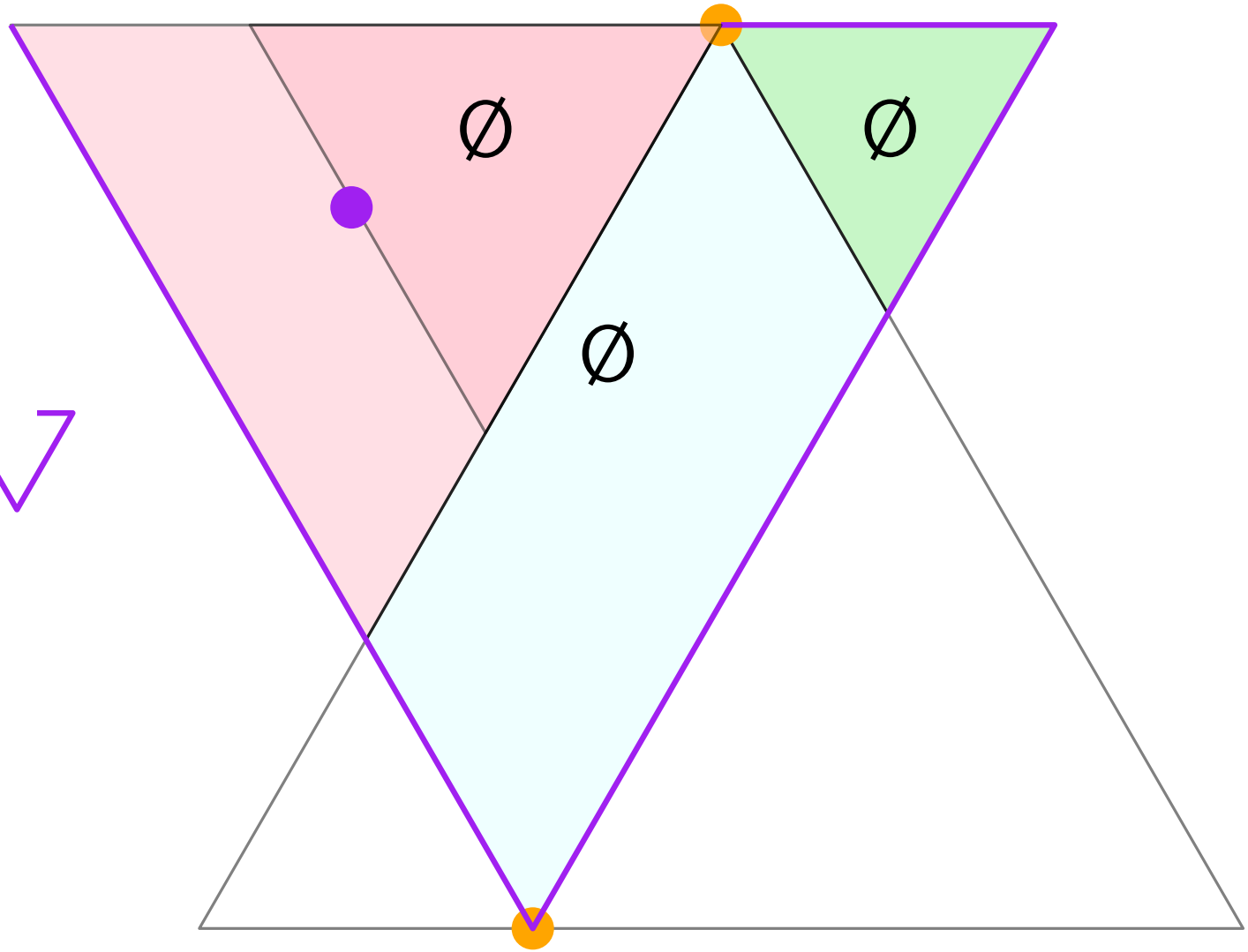
wlog  $\leq$



Length $\leq$ 

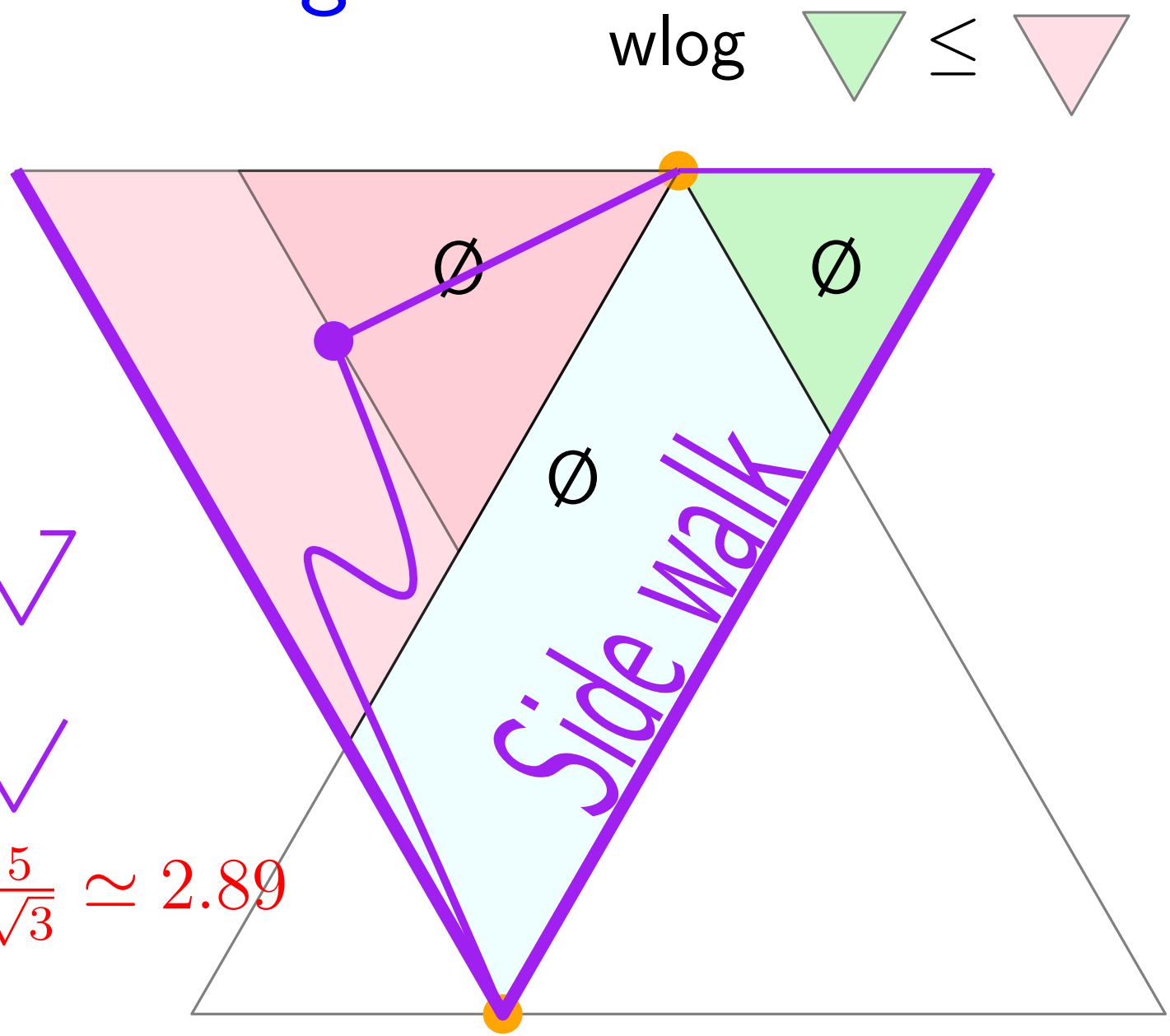
Negative routing

wlog  $\leq$



Length $\leq$

Negative routing

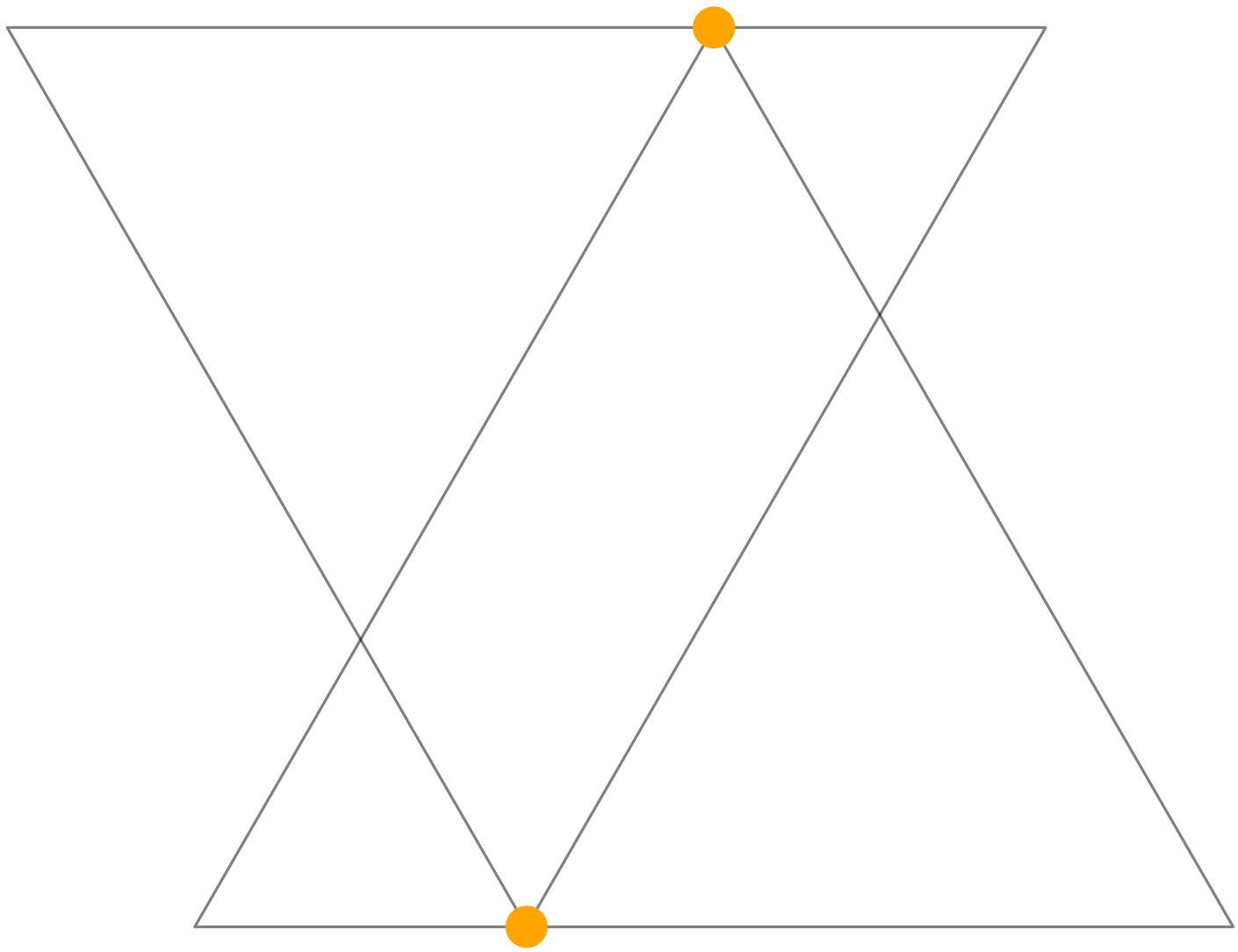


Length $\leq$

Length $\leq$

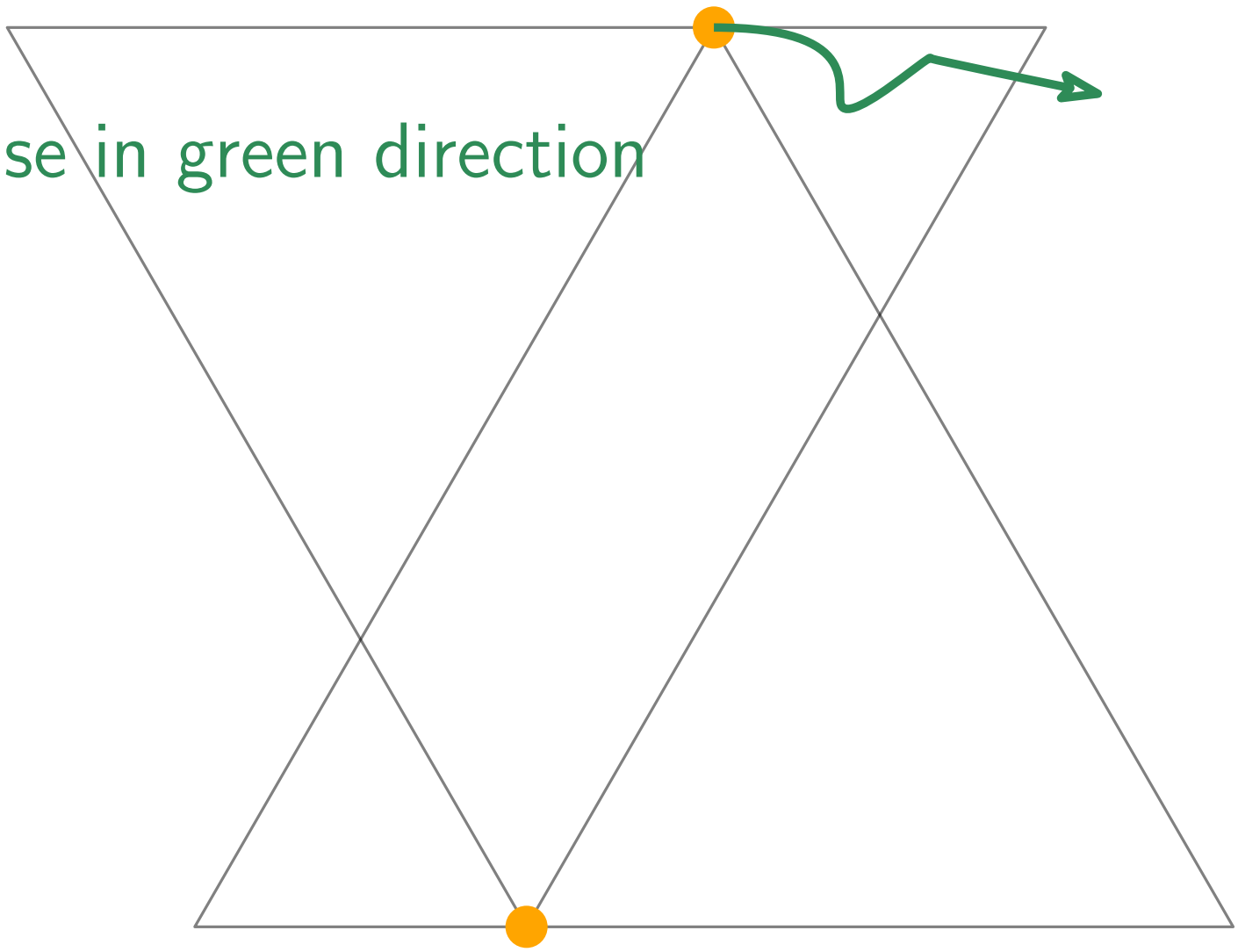
Stretch $\leq \frac{5}{\sqrt{3}} \simeq 2.89$

Memoryless negative routing



Memoryless negative routing

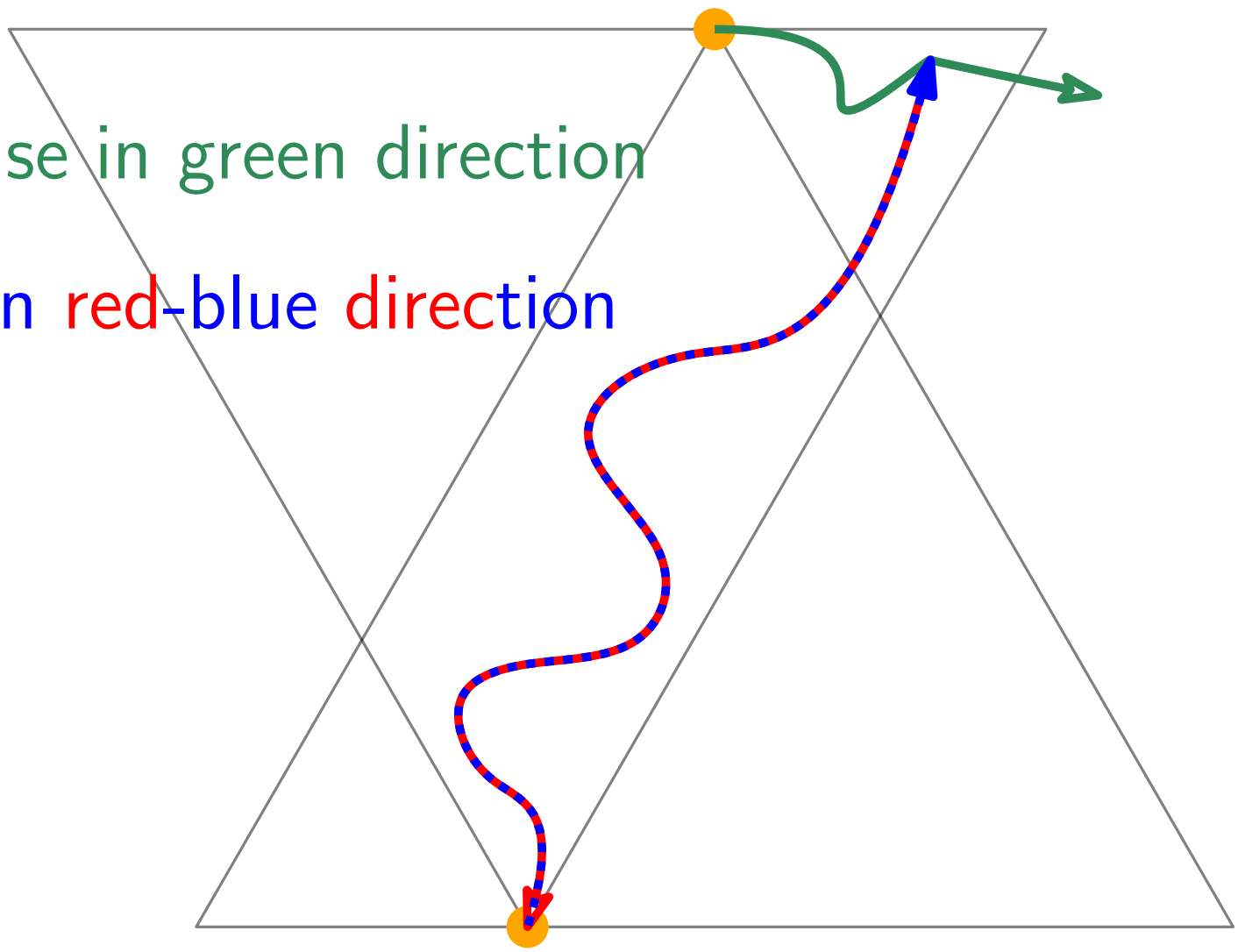
forward phase in green direction



Memoryless negative routing

forward phase in green direction

side phase in red-blue direction

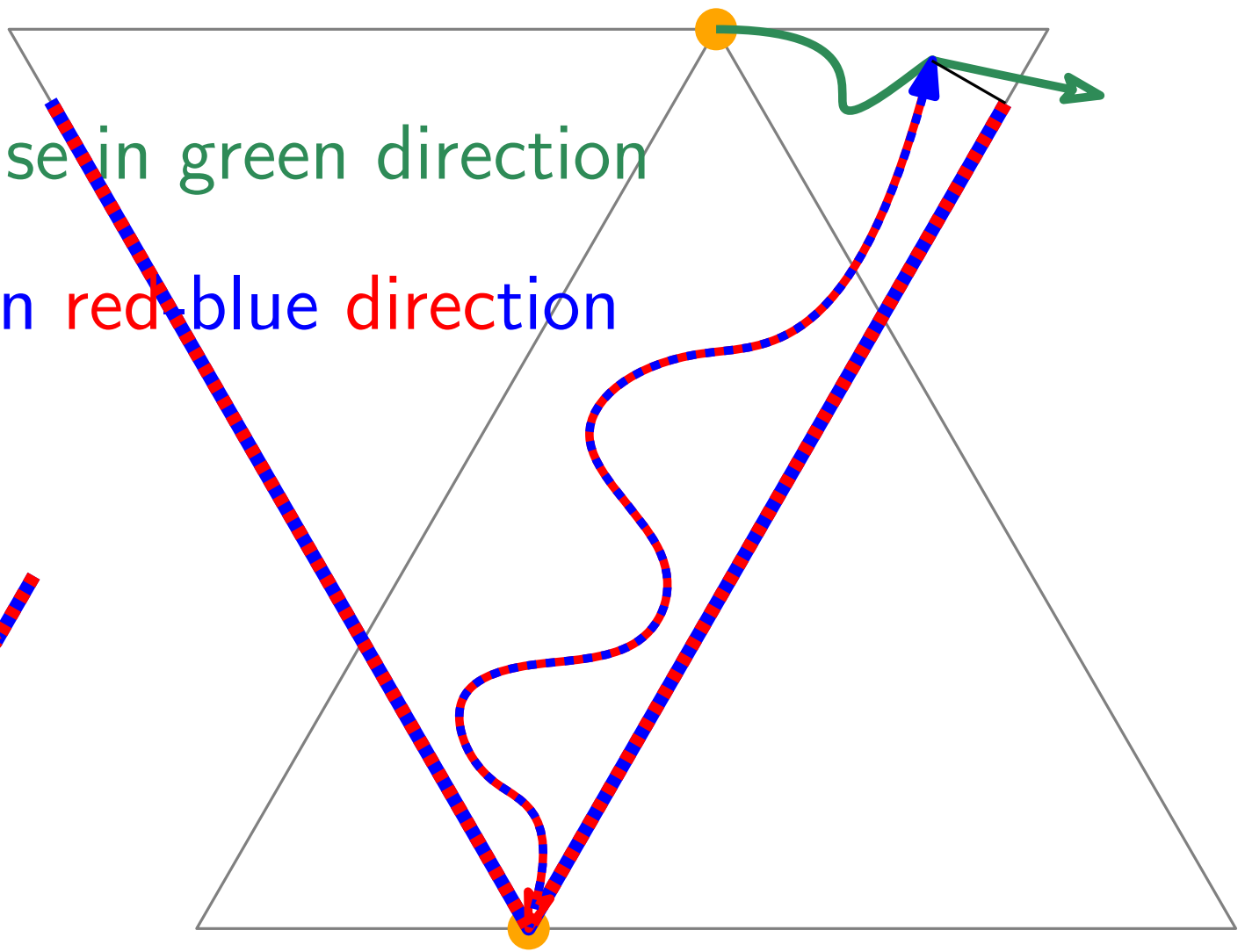


Memoryless negative routing

forward phase in green direction

side phase in red-blue direction

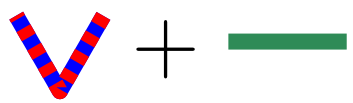
Length $\leq$ 



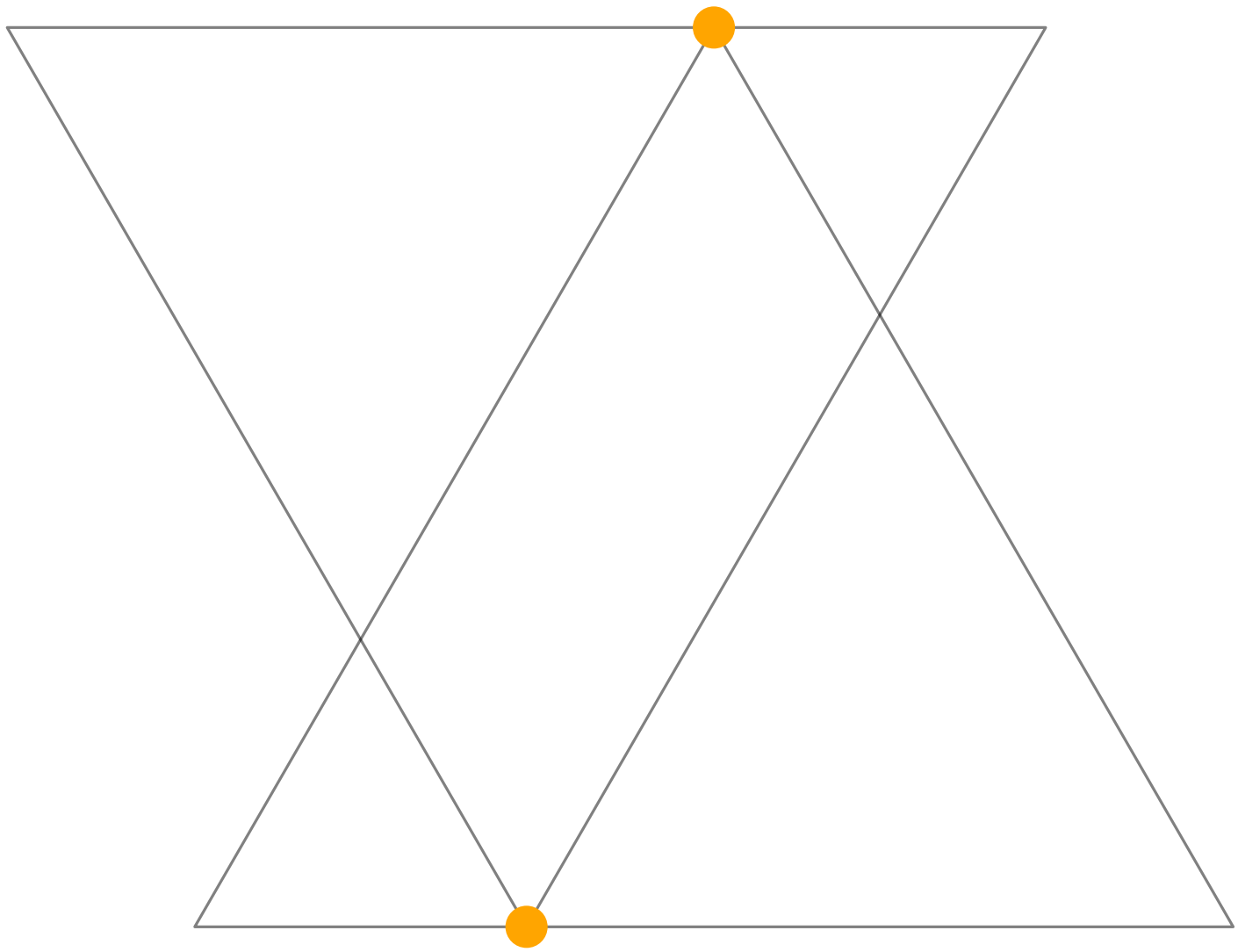
Memoryless negative routing

forward phase in green direction

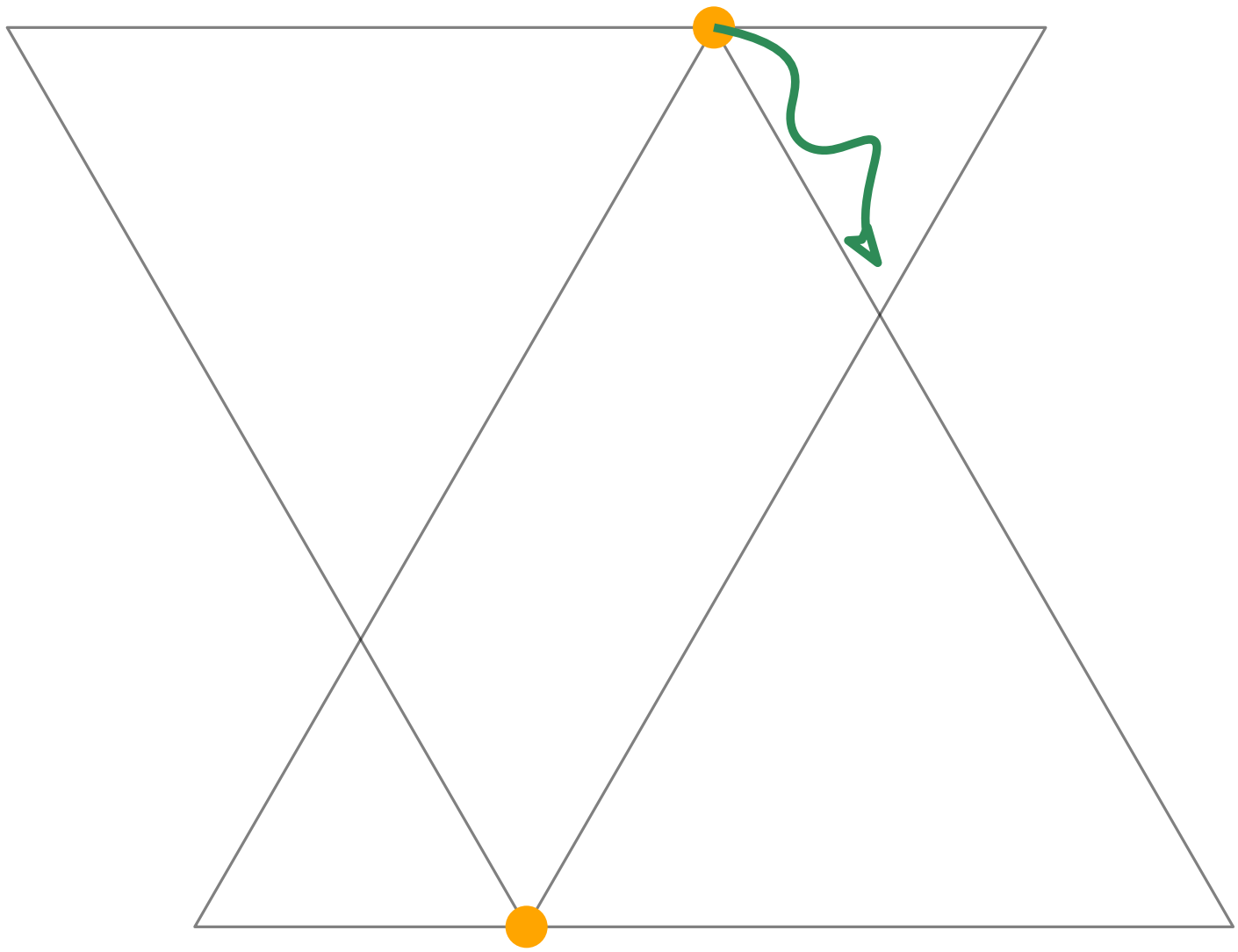
side phase in red-blue direction

Length $\leq$  $+$

1-Memory negative routing



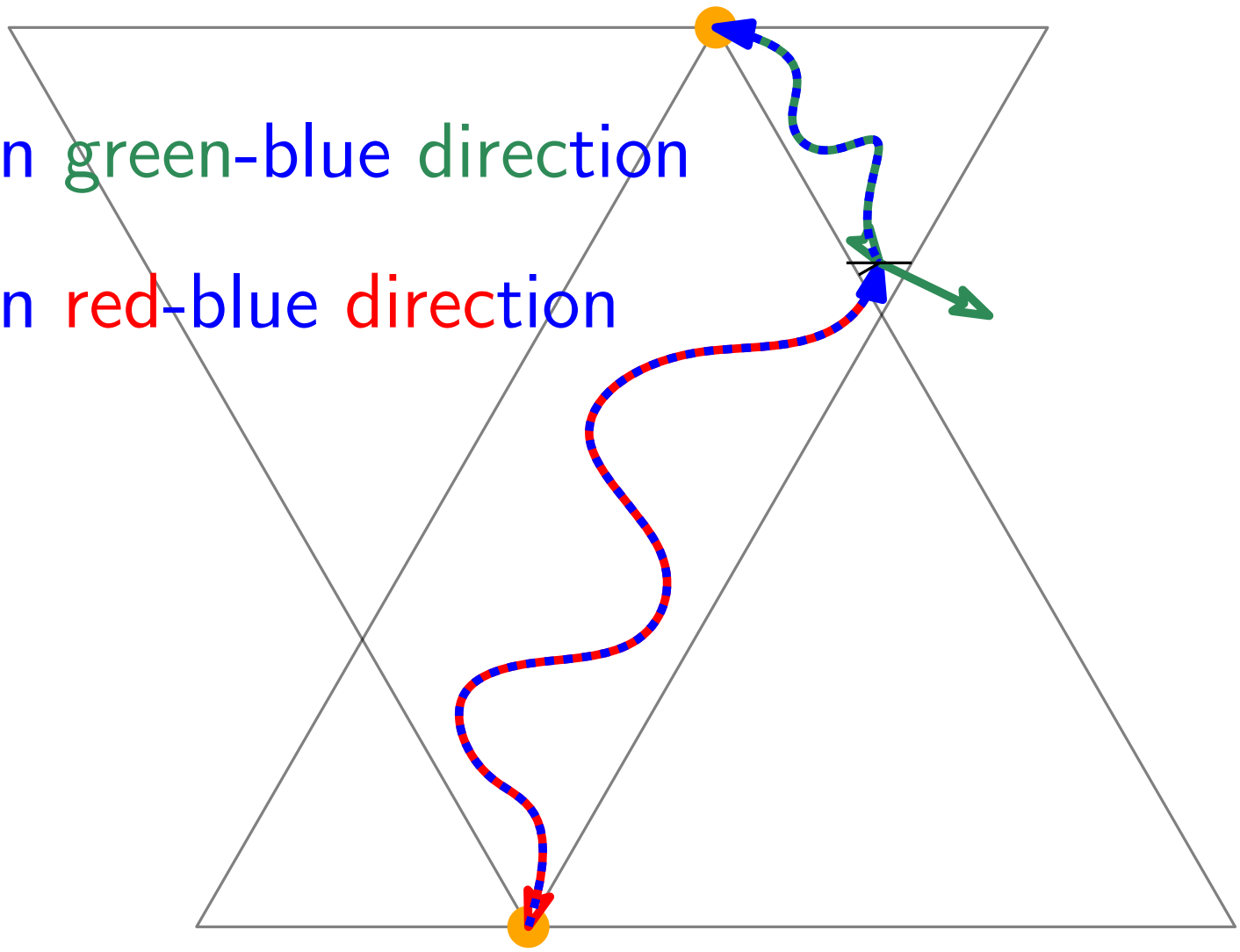
1-Memory negative routing



1-Memory negative routing

side phase in green-blue direction

side phase in red-blue direction



1-Memory negative routing

side phase in green-blue direction

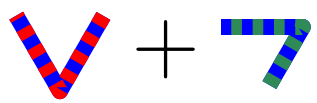
side phase in red-blue direction

Length $\leq$ 

1-Memory negative routing

side phase in green-blue direction

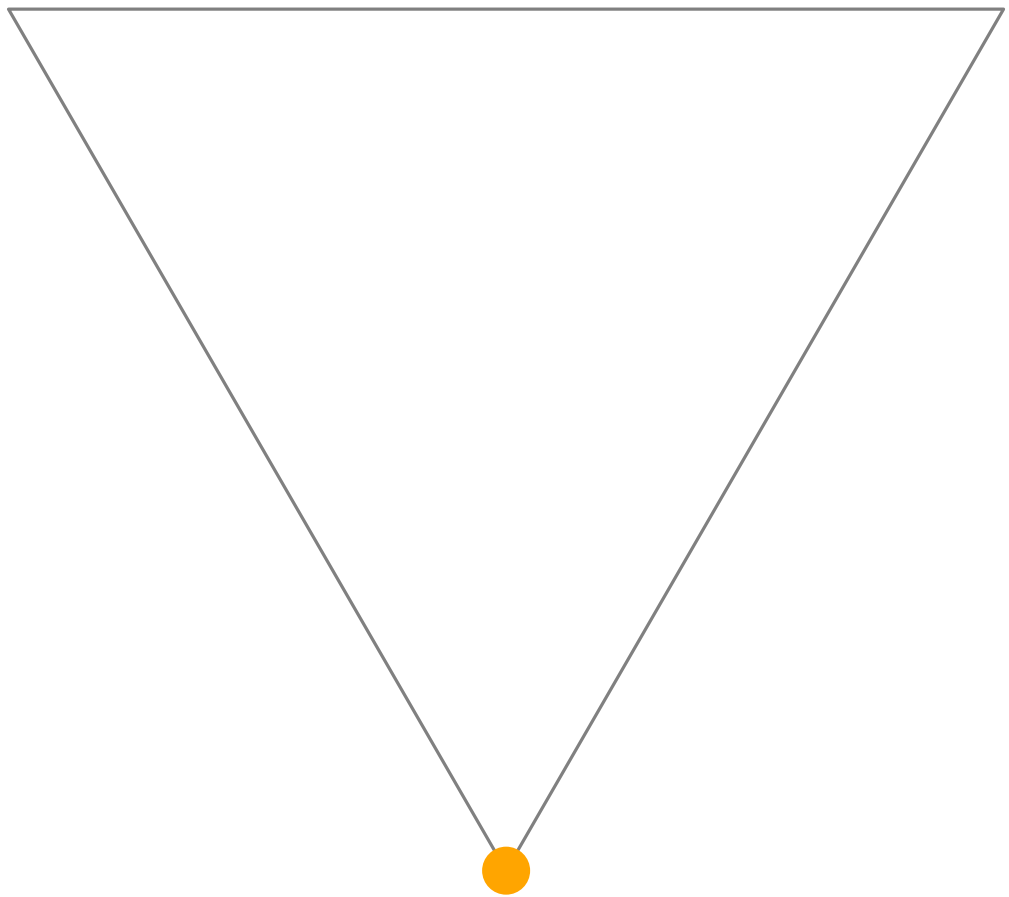
side phase in red-blue direction

Length $\leq$  $+$

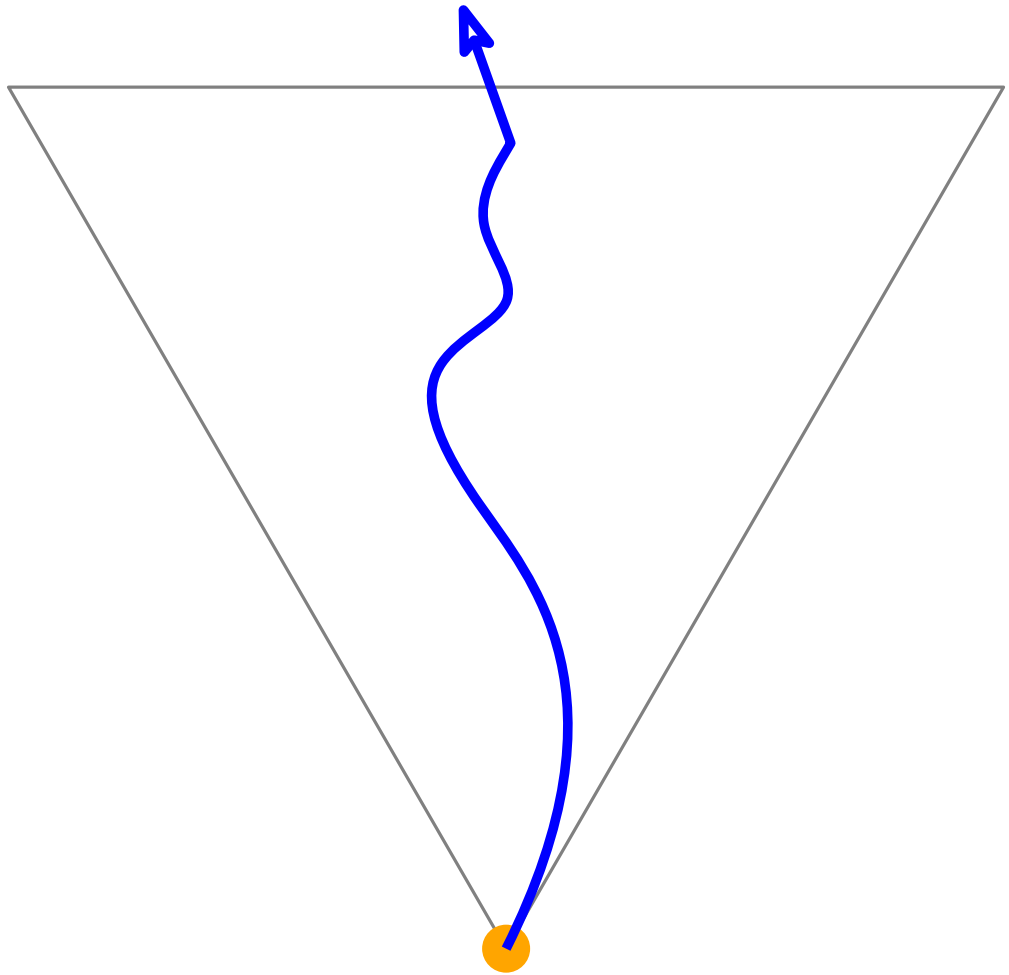
Probabilistic analysis

Expected length ?

Forward routing

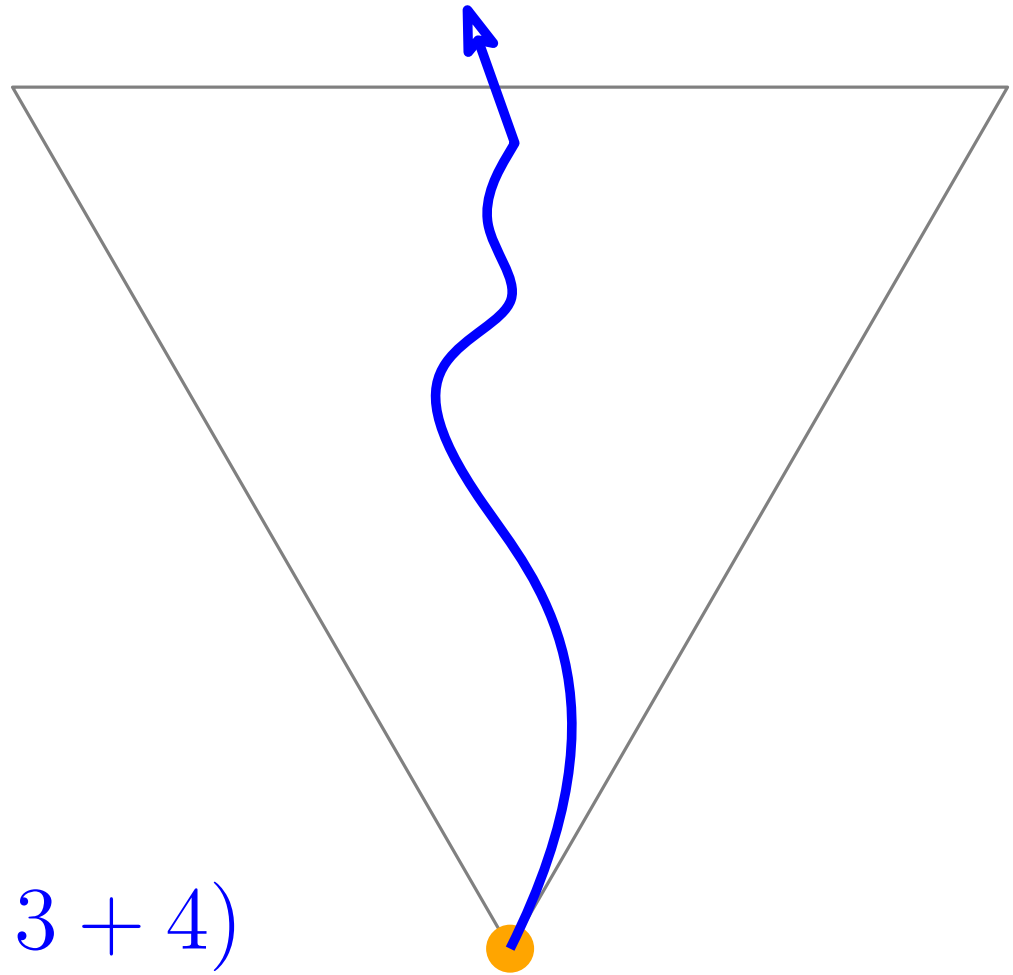


Forward routing



With high probability

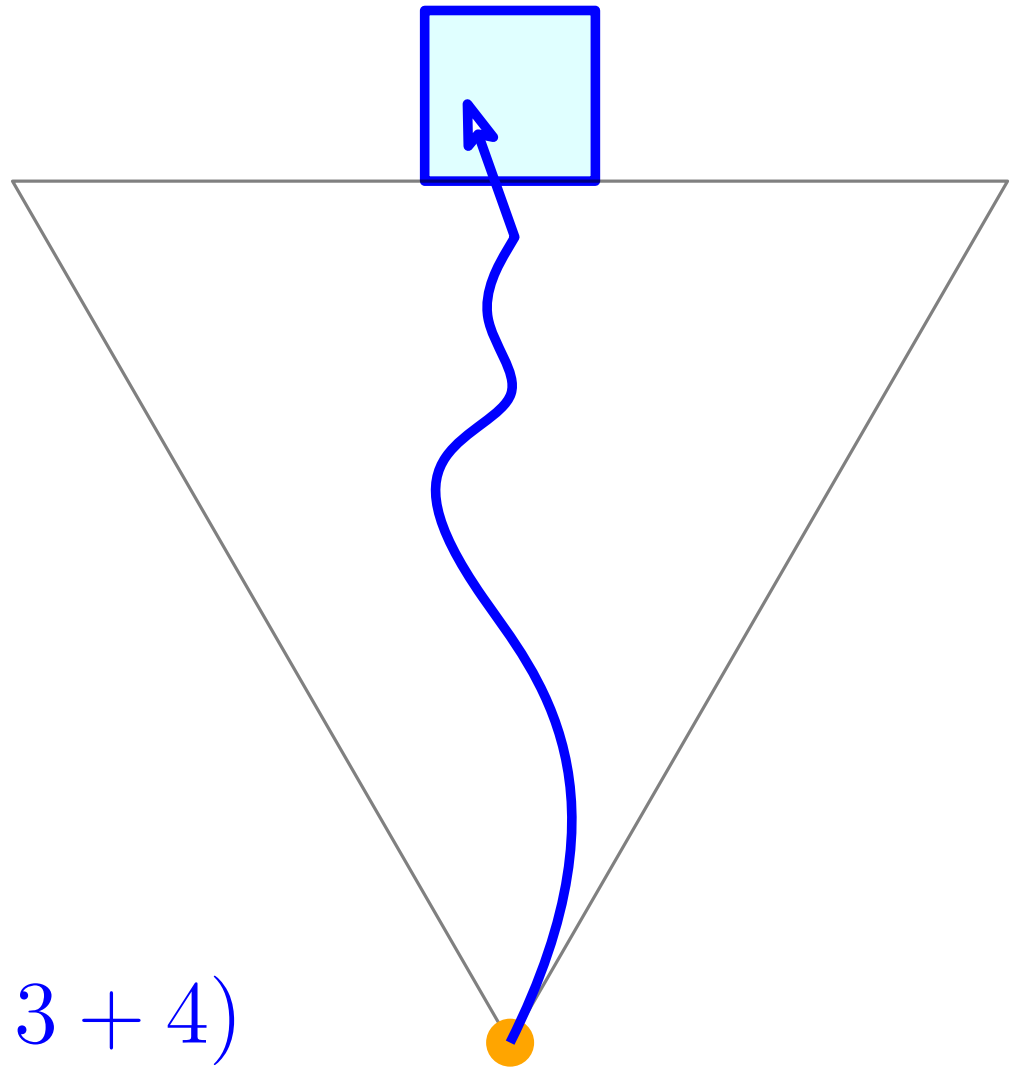
Forward routing



With high probability

stretch close to $\frac{\sqrt{3}}{12}(3\ln 3 + 4)$

Forward routing



With high probability

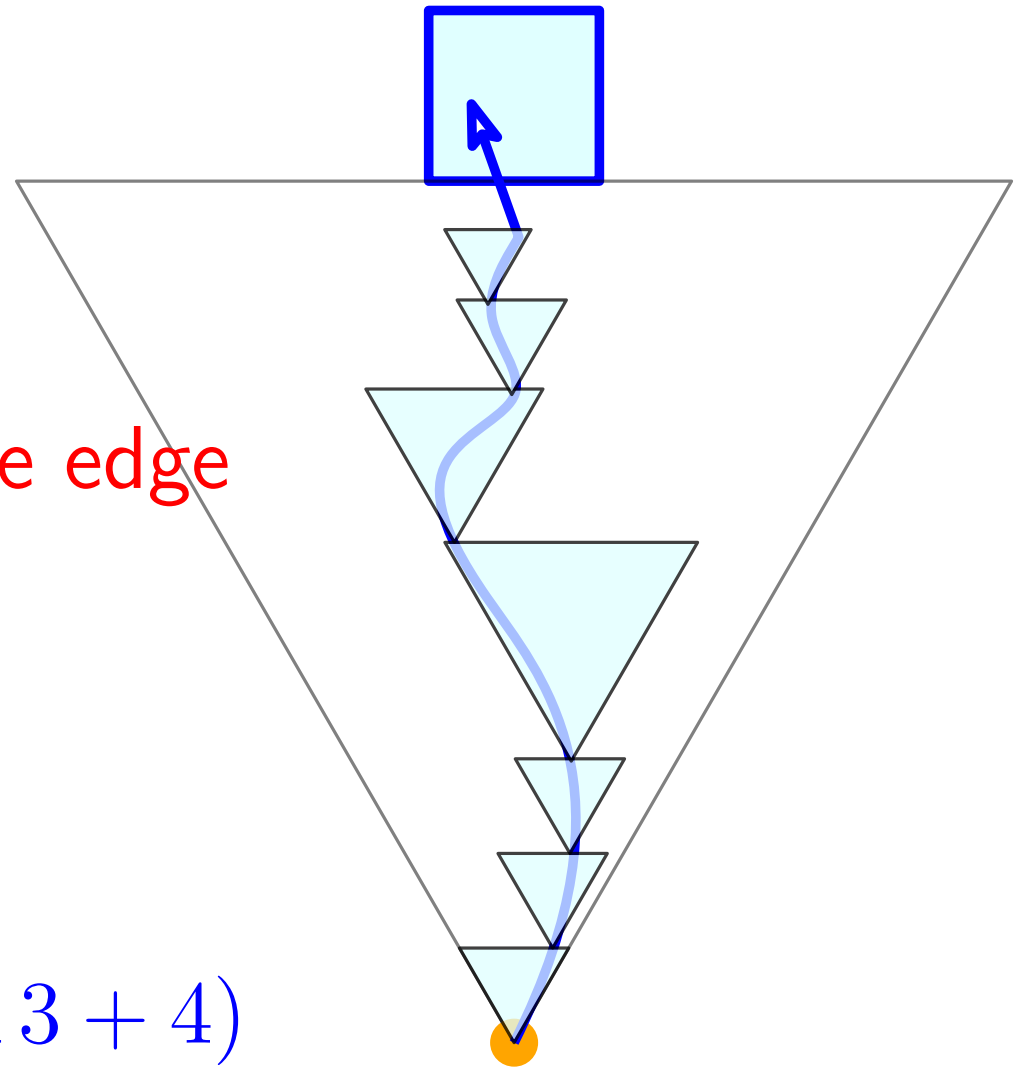
stretch close to $\frac{\sqrt{3}}{12}(3\ln 3 + 4)$

path terminates in a square of side $\tilde{O}(\lambda^{\frac{1}{4}})$

Forward routing

Proof:

- Independance
- Expected slope of one edge

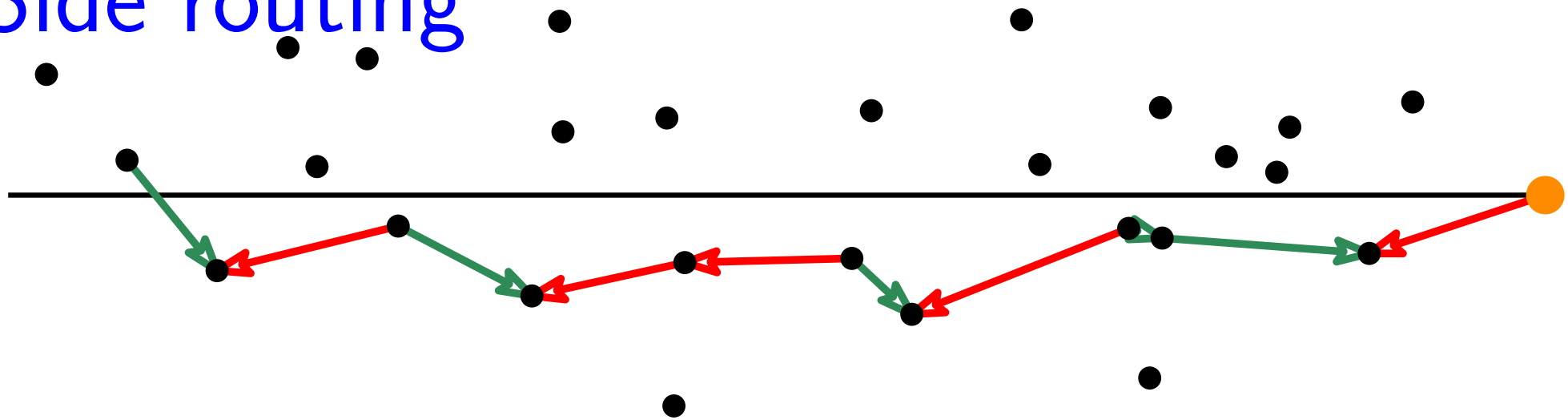


With high probability

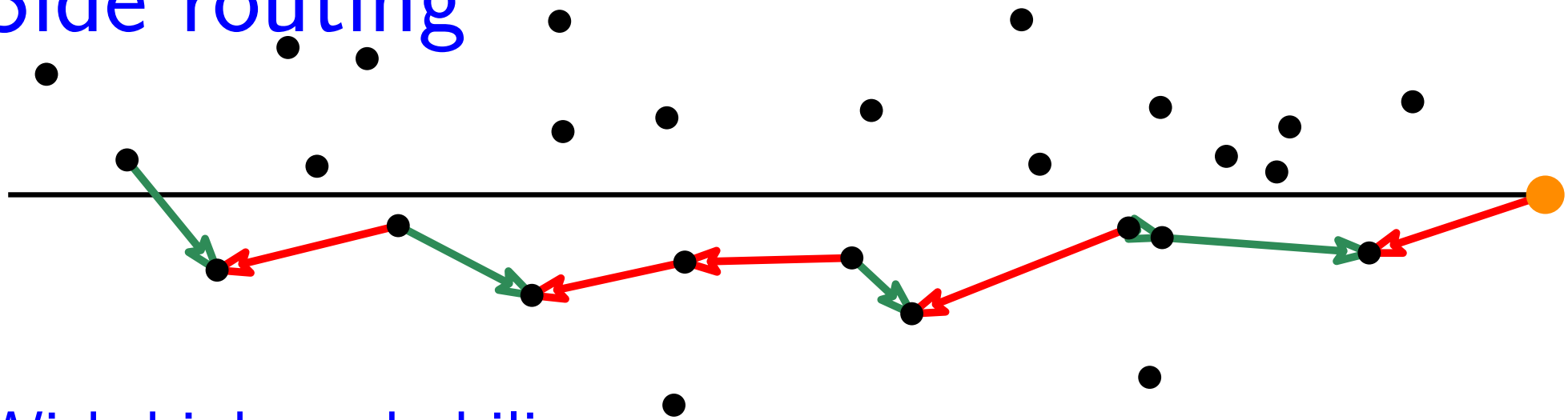
stretch close to $\frac{\sqrt{3}}{12}(3\ln 3 + 4)$

path terminates in a square of side $\tilde{O}(\lambda^{\frac{1}{4}})$

Side routing

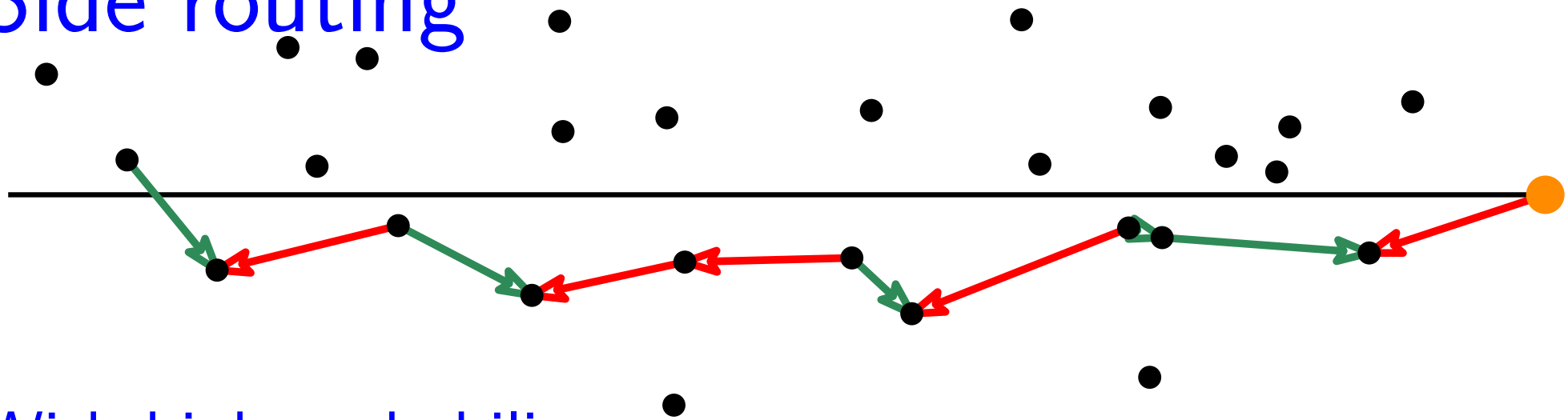


Side routing



With high probability

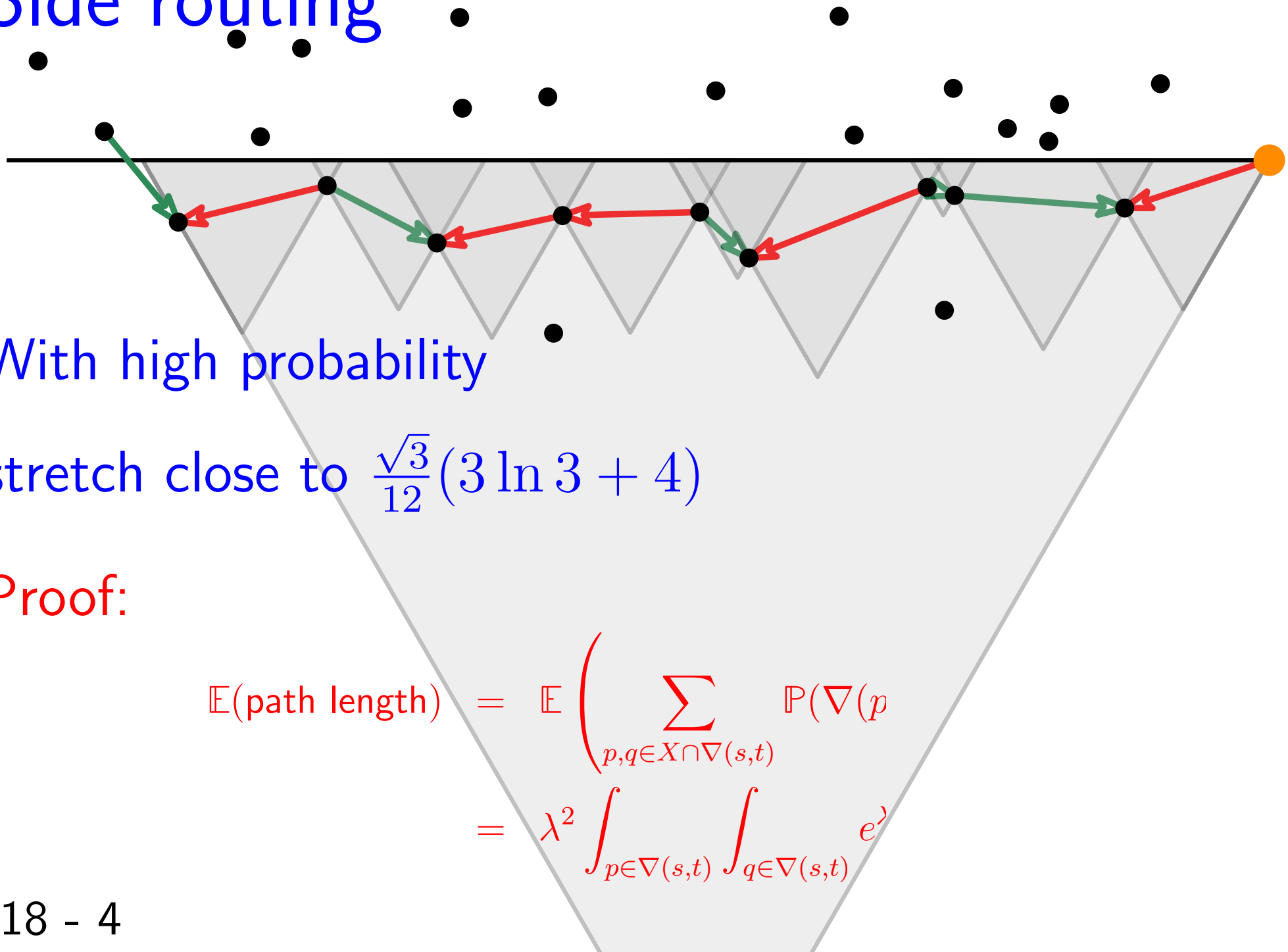
Side routing



With high probability

stretch close to $\frac{\sqrt{3}}{12}(3\ln 3 + 4)$

Side routing



With high probability

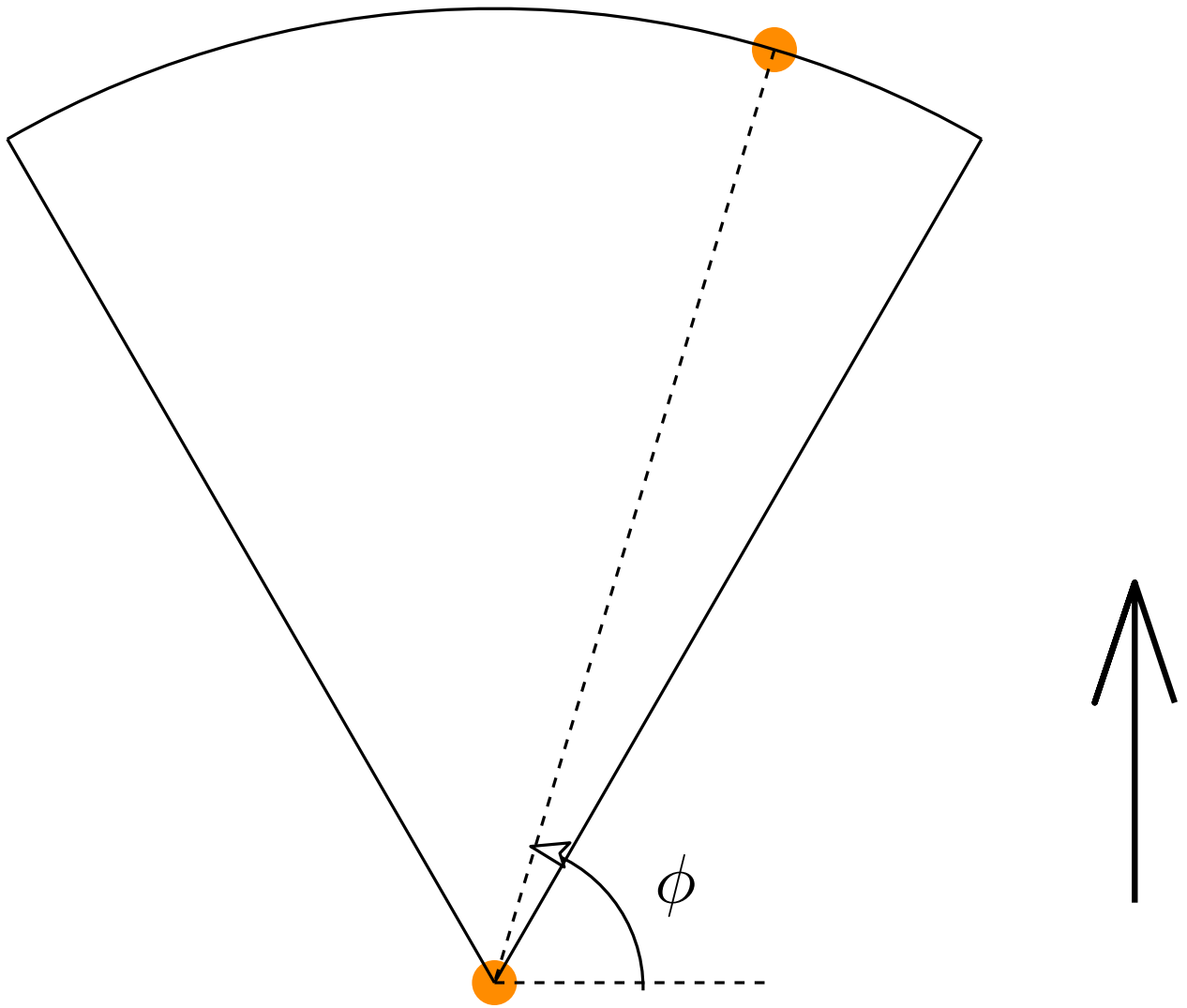
stretch close to $\frac{\sqrt{3}}{12}(3\ln 3 + 4)$

Proof:

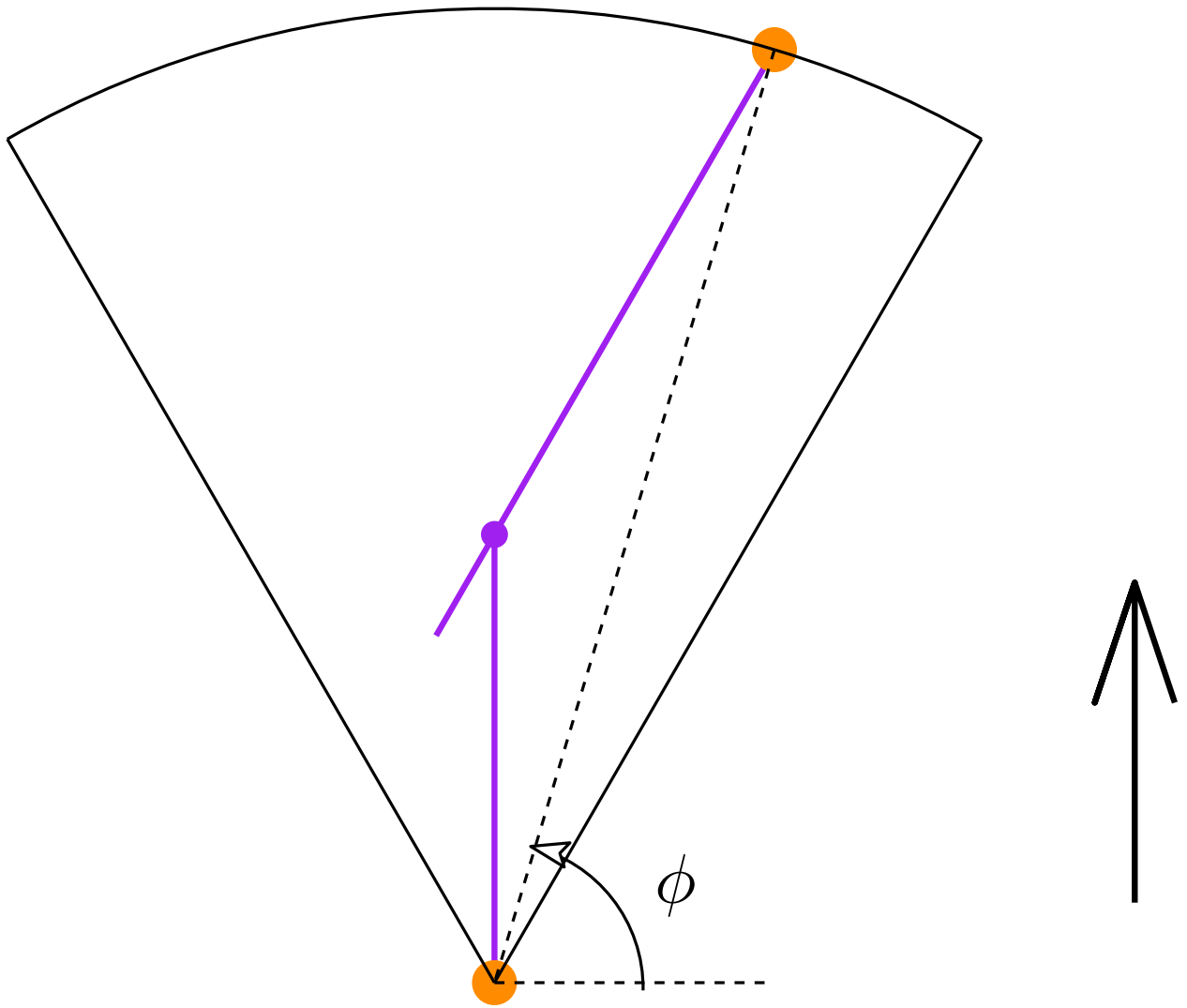
$$\mathbb{E}(\text{path length}) = \mathbb{E}\left(\sum_{p,q\in X\cap\nabla(s,t)} \mathbb{P}(\nabla(p\right.$$

$$= \lambda^2 \int_{p\in\nabla(s,t)} \int_{q\in\nabla(s,t)} e^{\lambda}$$

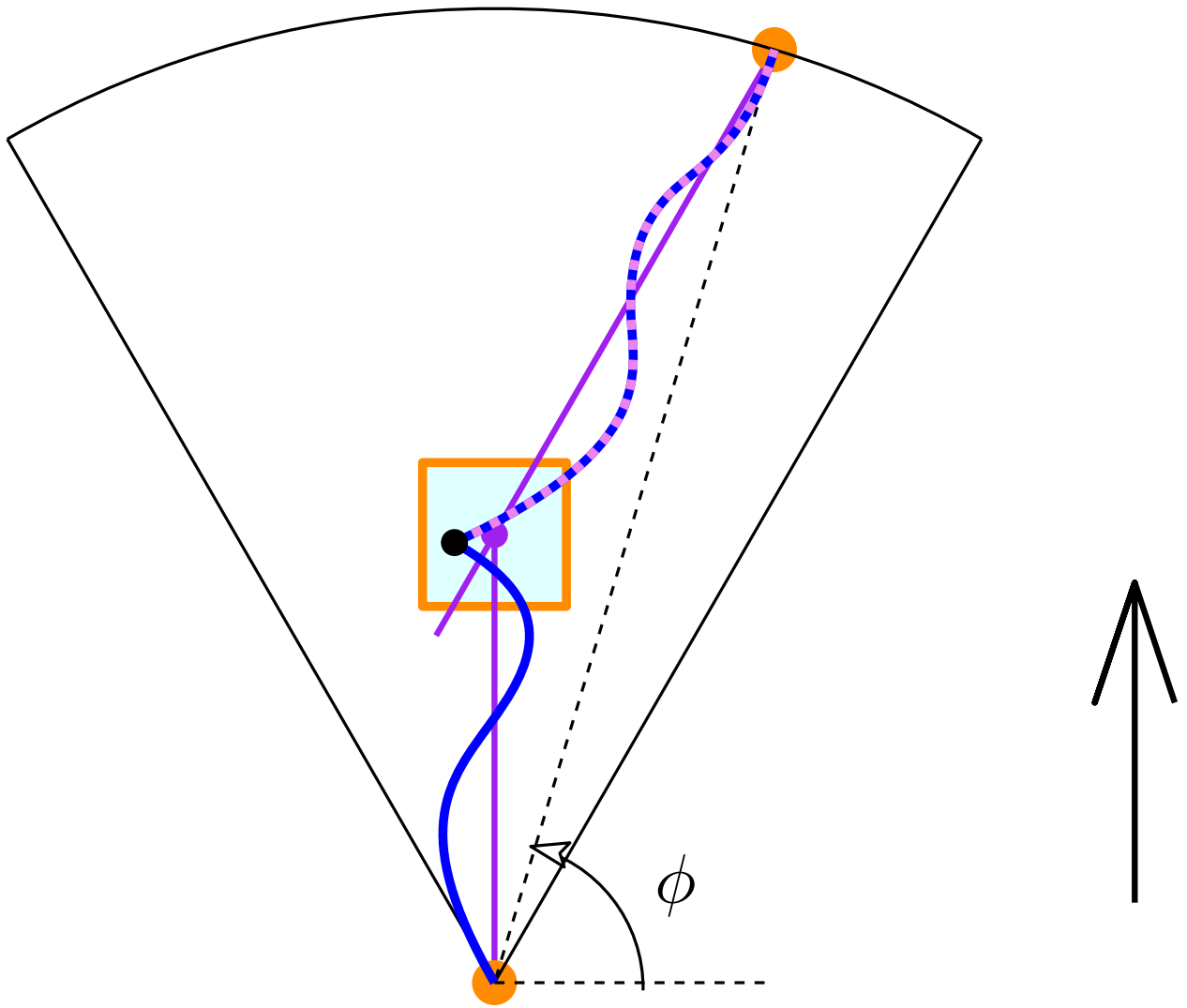
Θ_6 routing



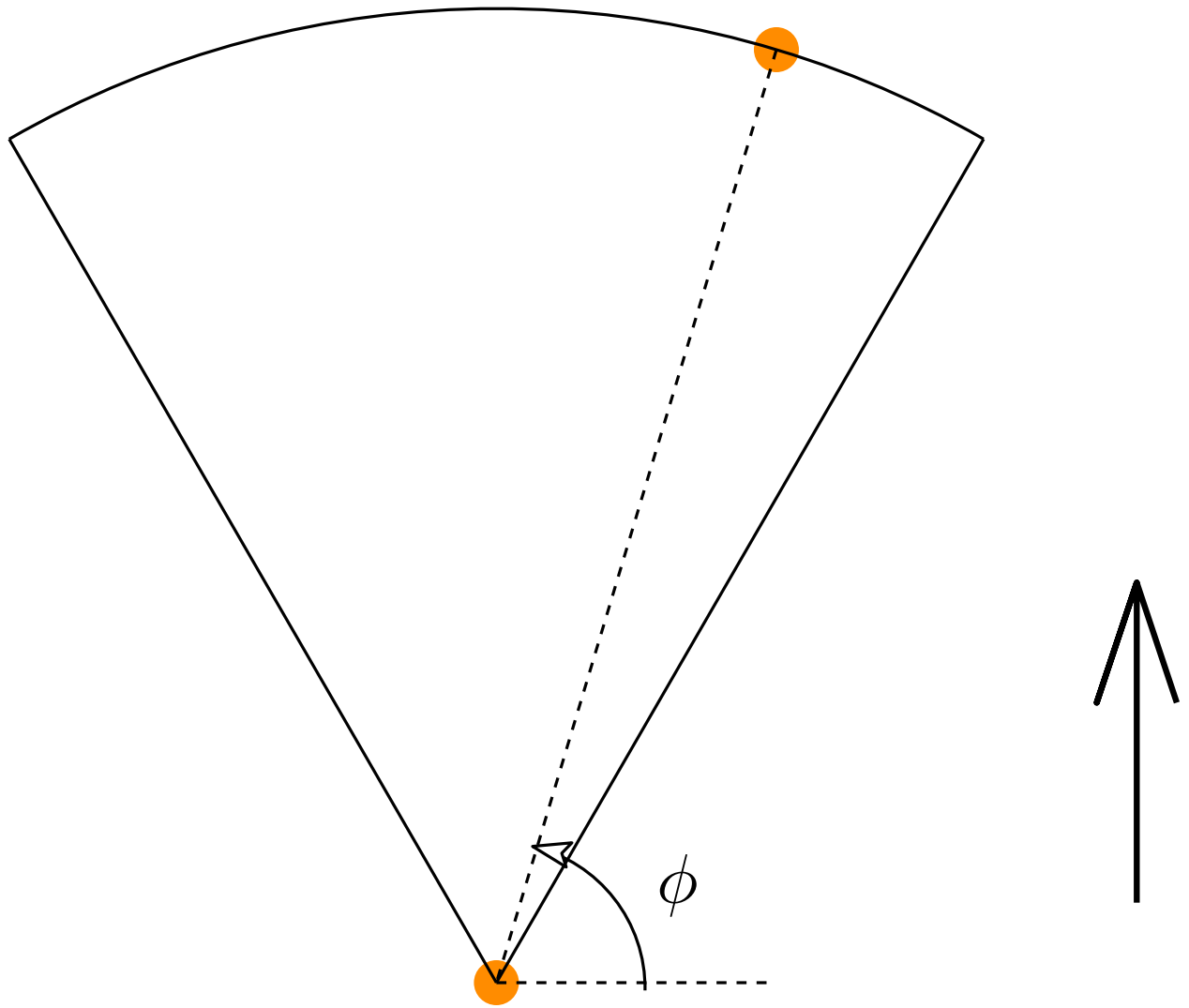
Θ_6 routing



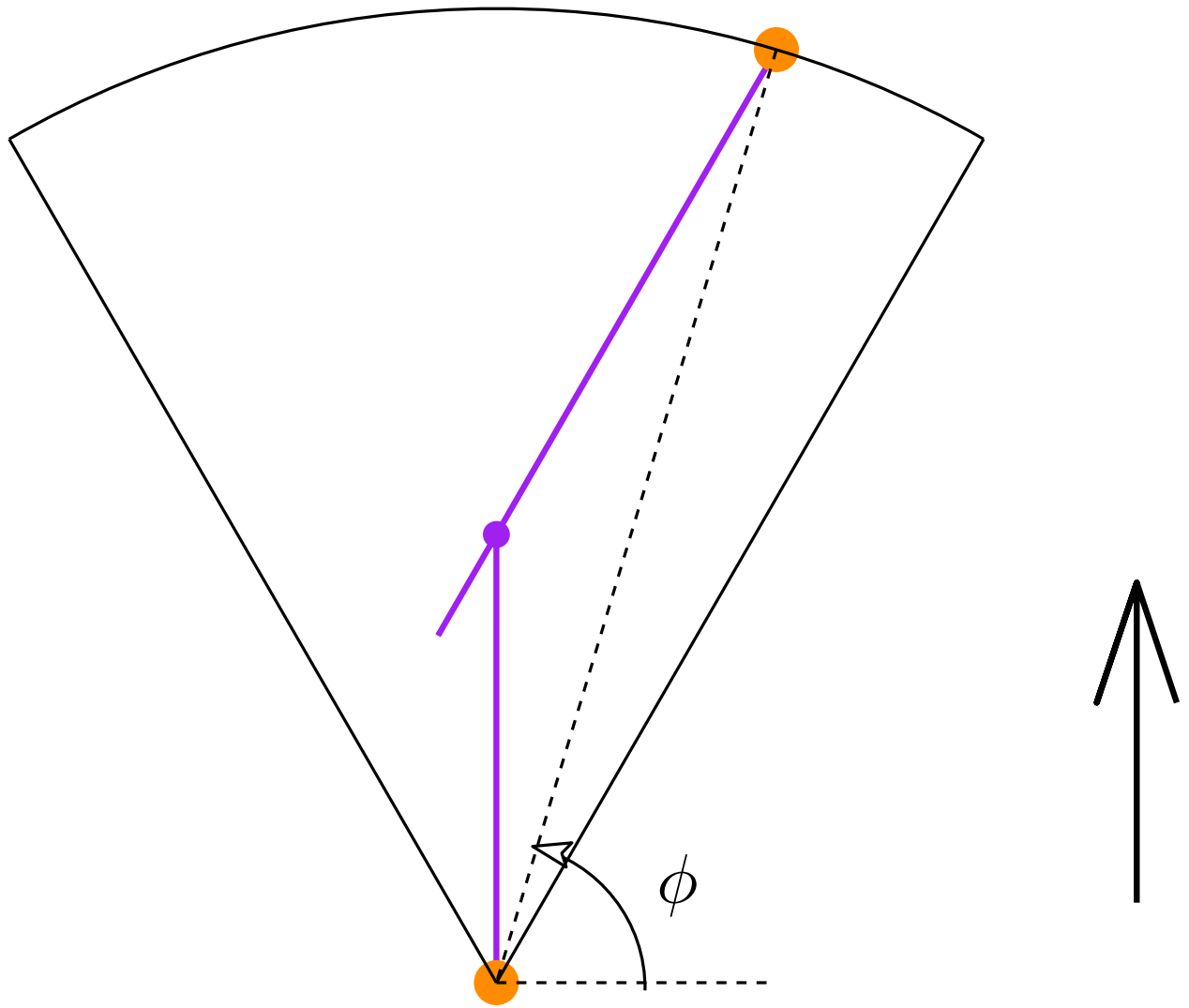
Θ_6 routing



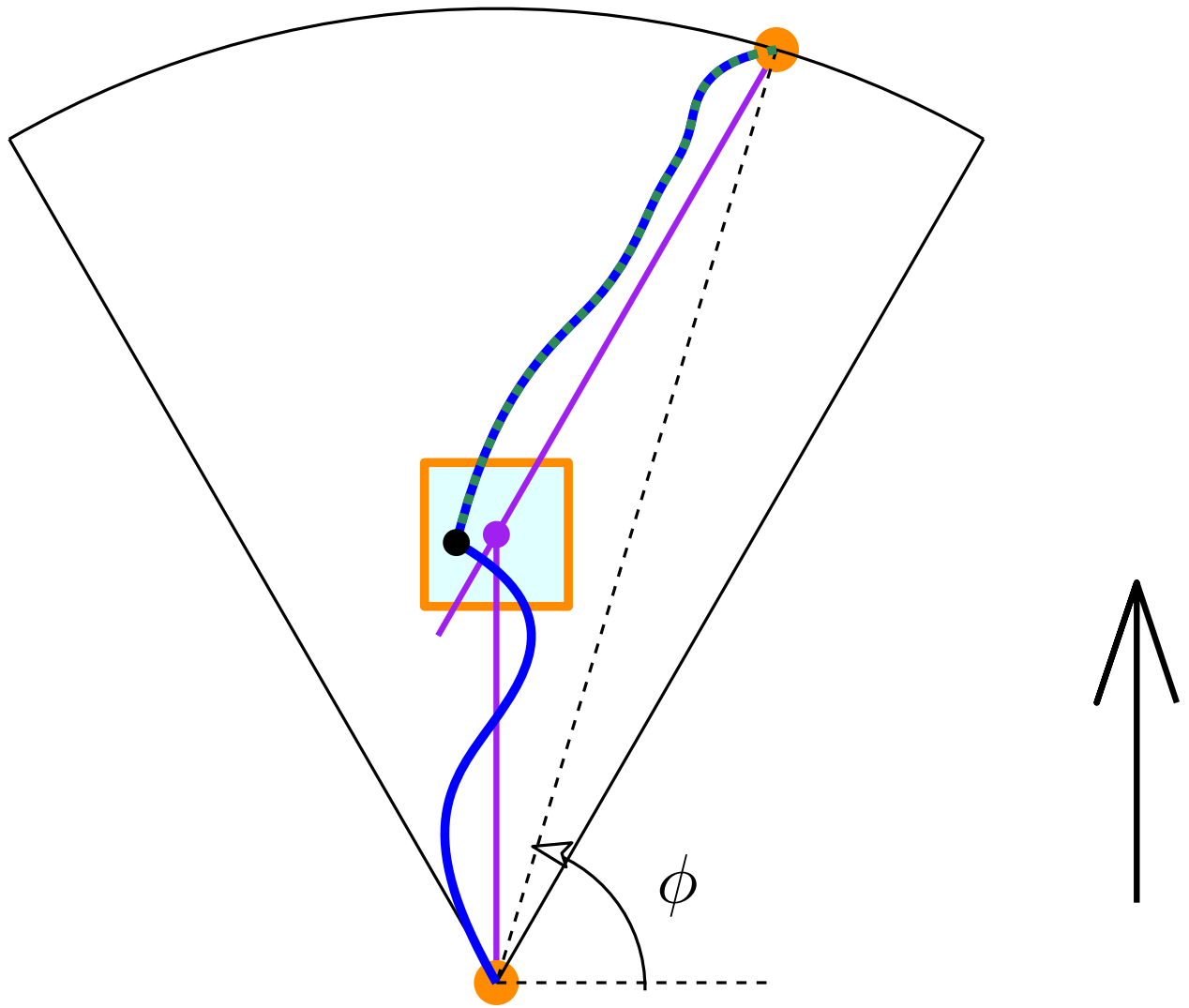
Positive routing



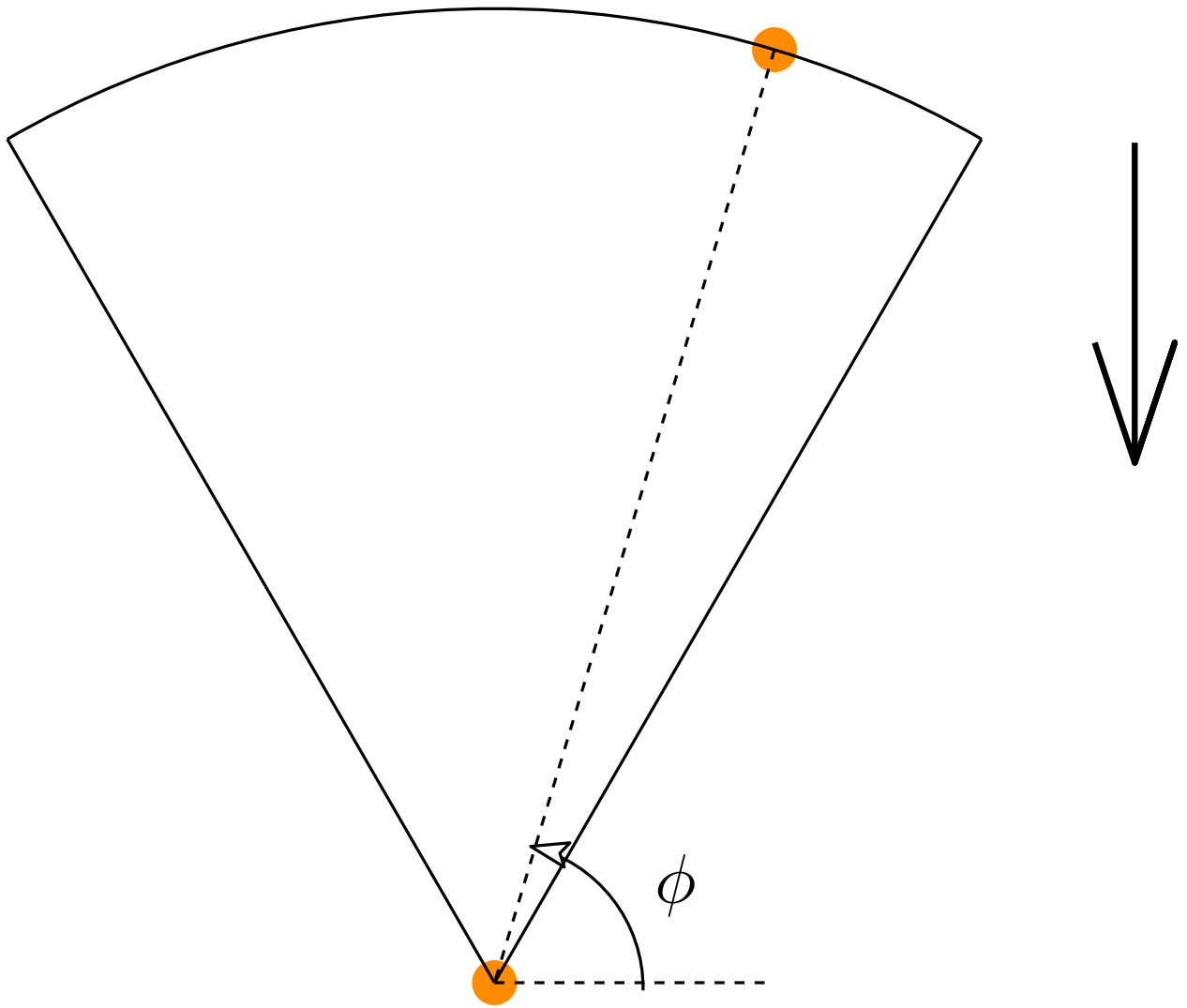
Positive routing



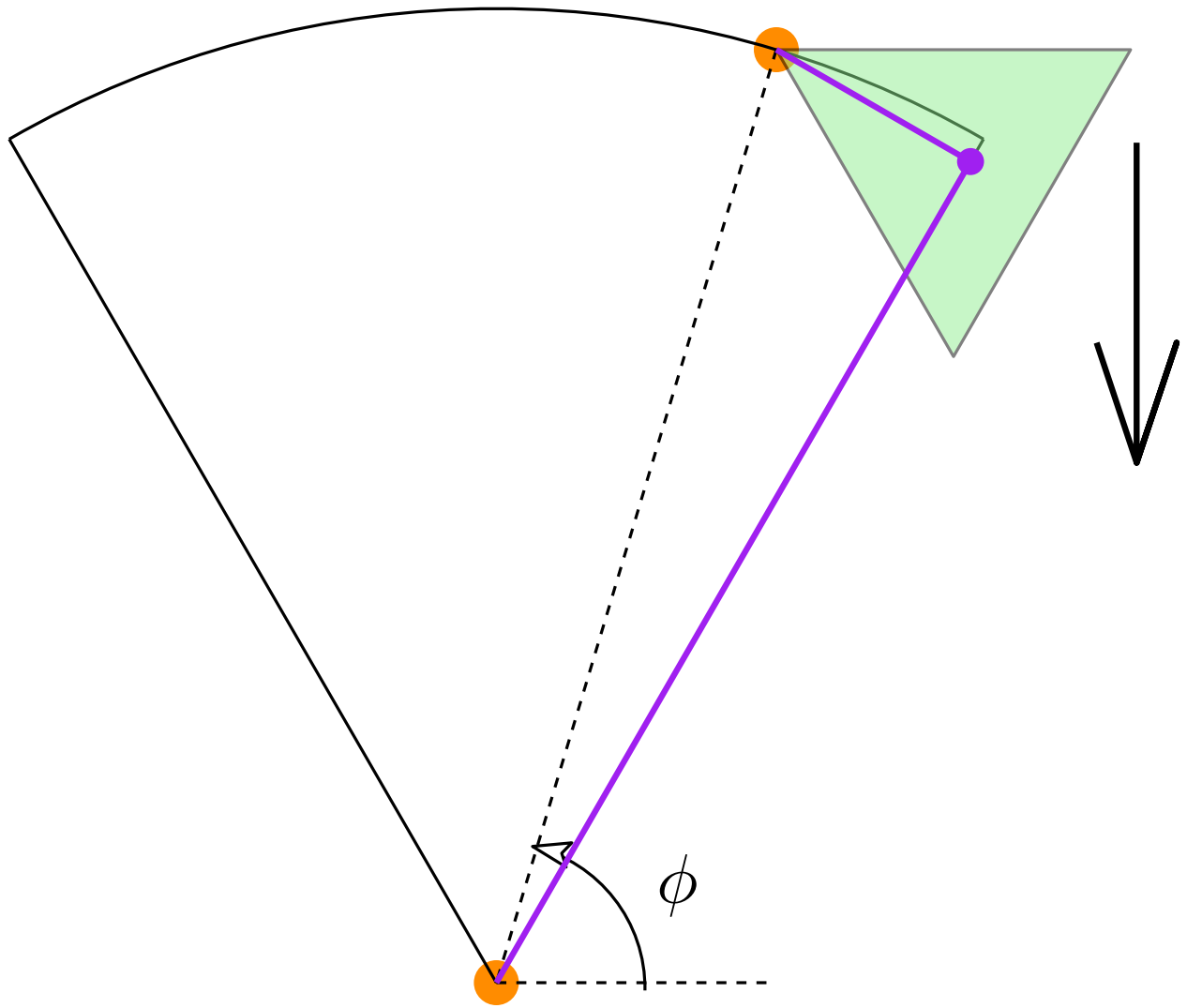
Positive routing



Memoryless negative routing

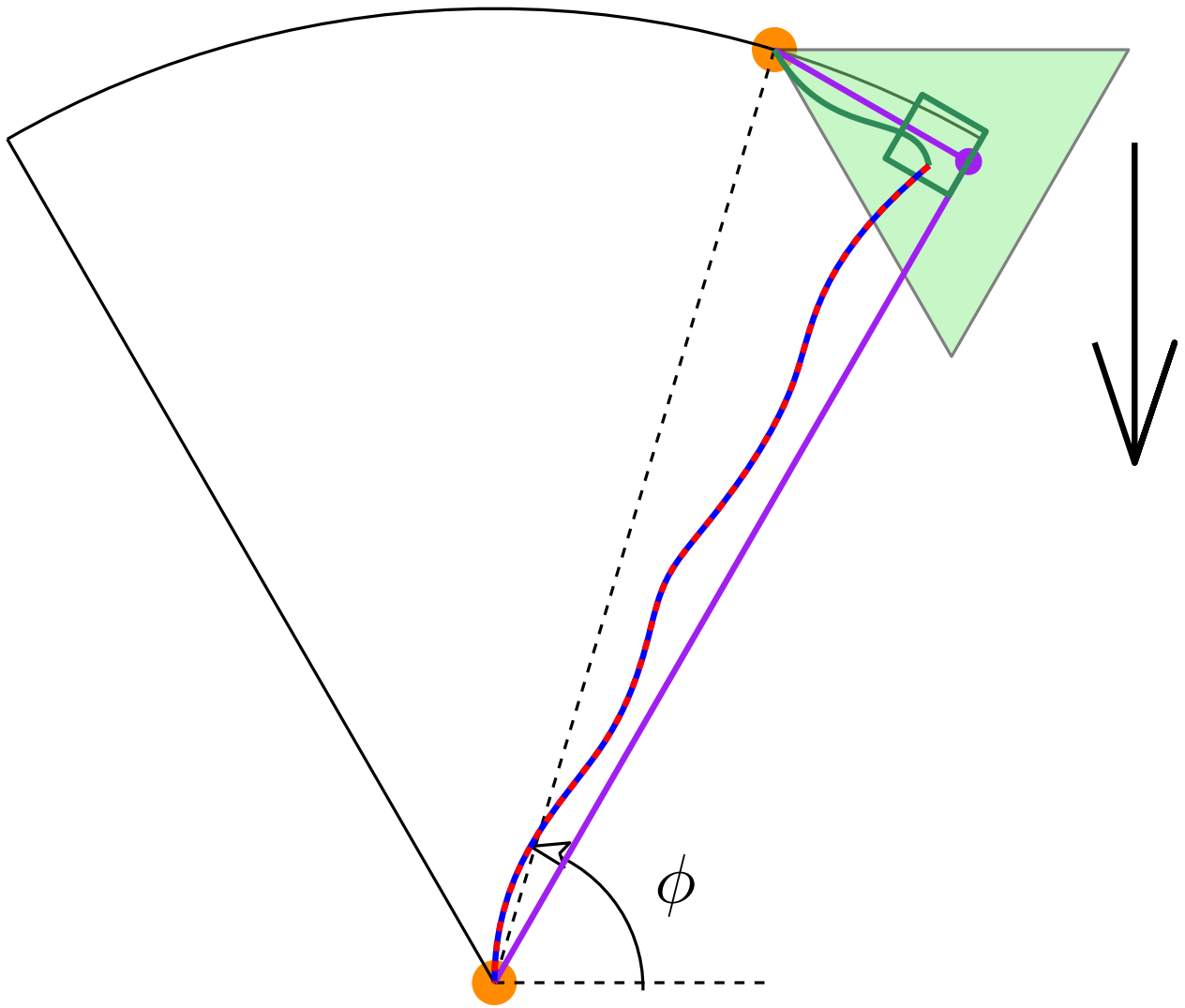


Memoryless negative routing

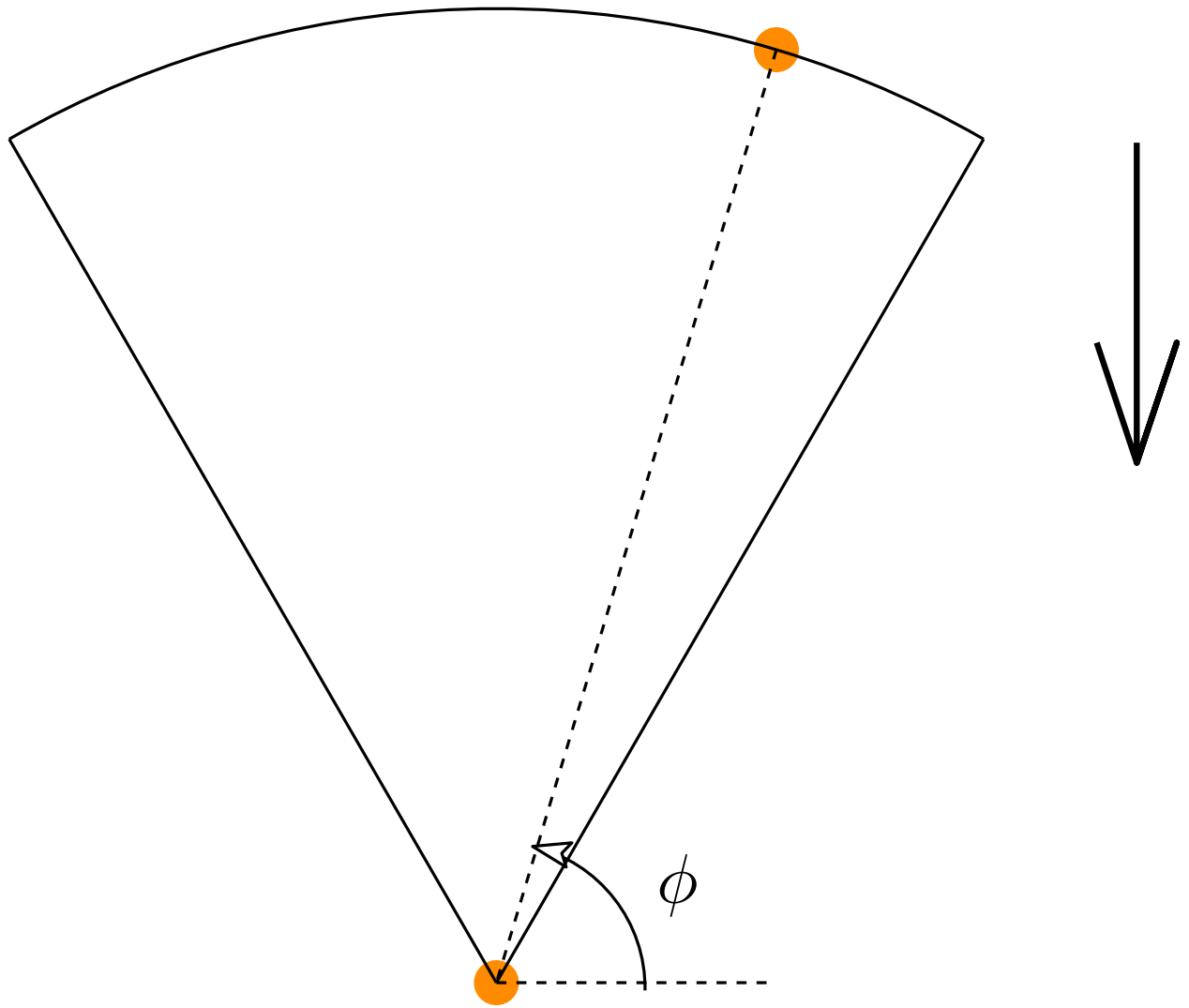


Memoryless negative routing

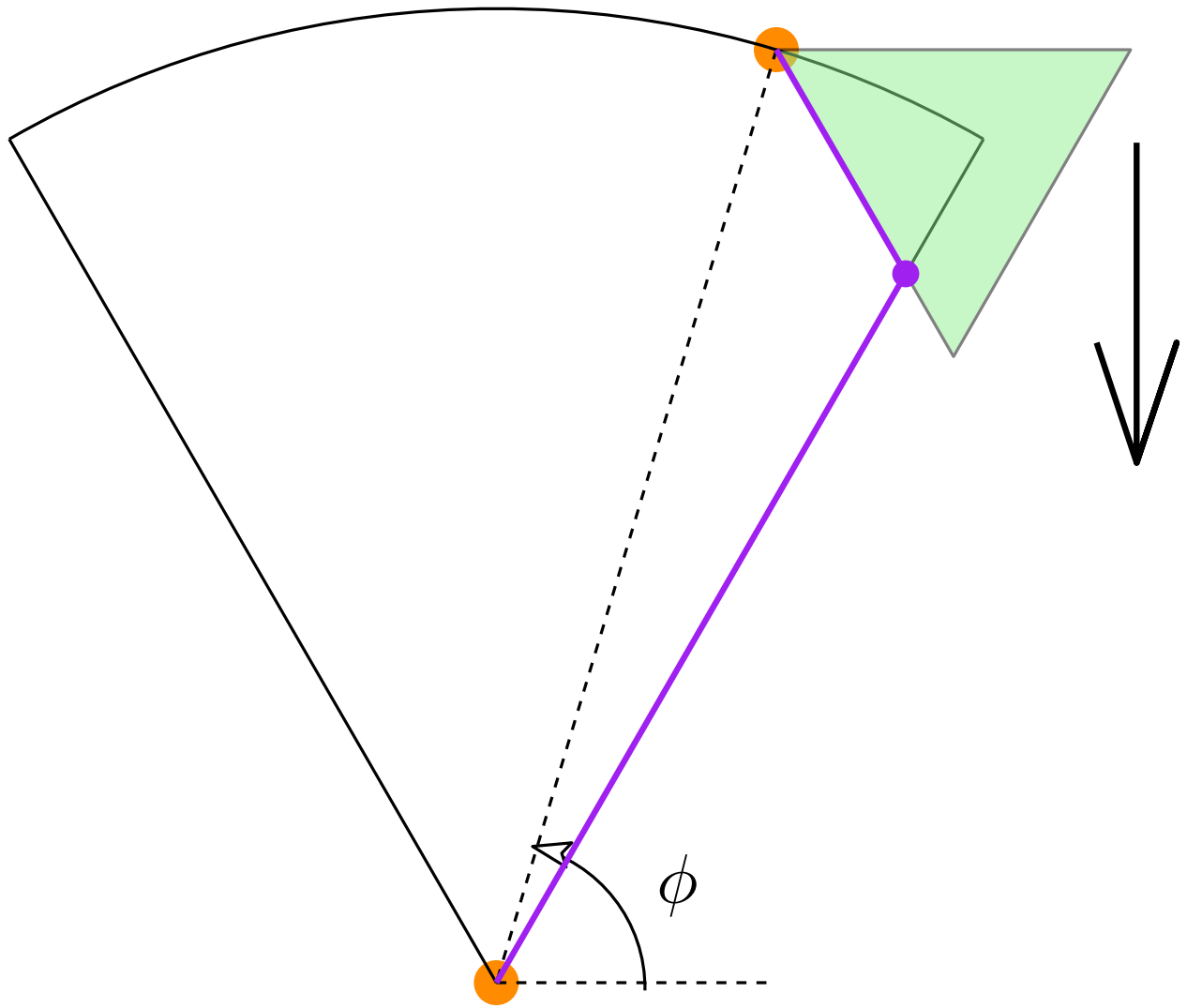
Forward + Side



1-Memory negative routing

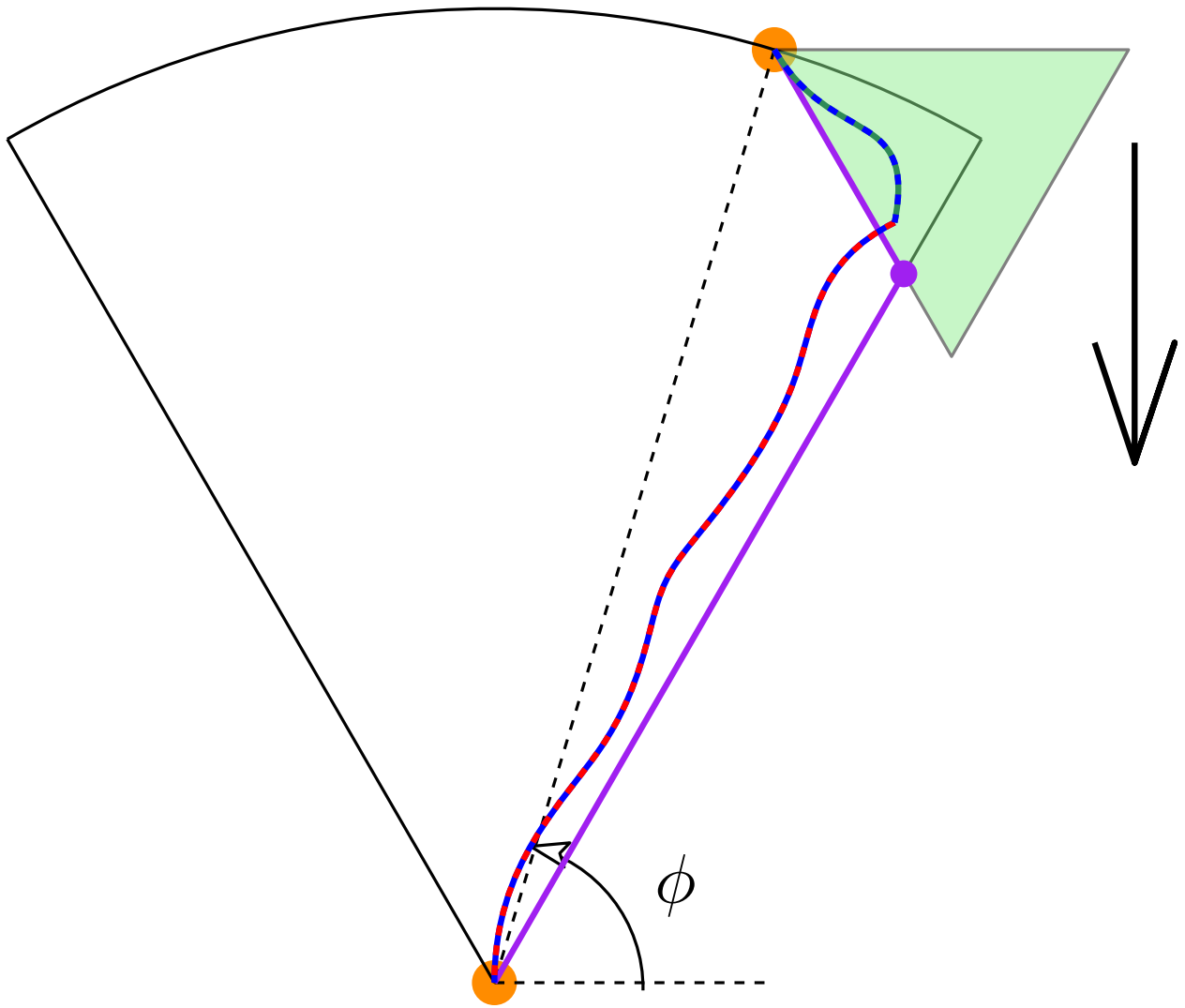


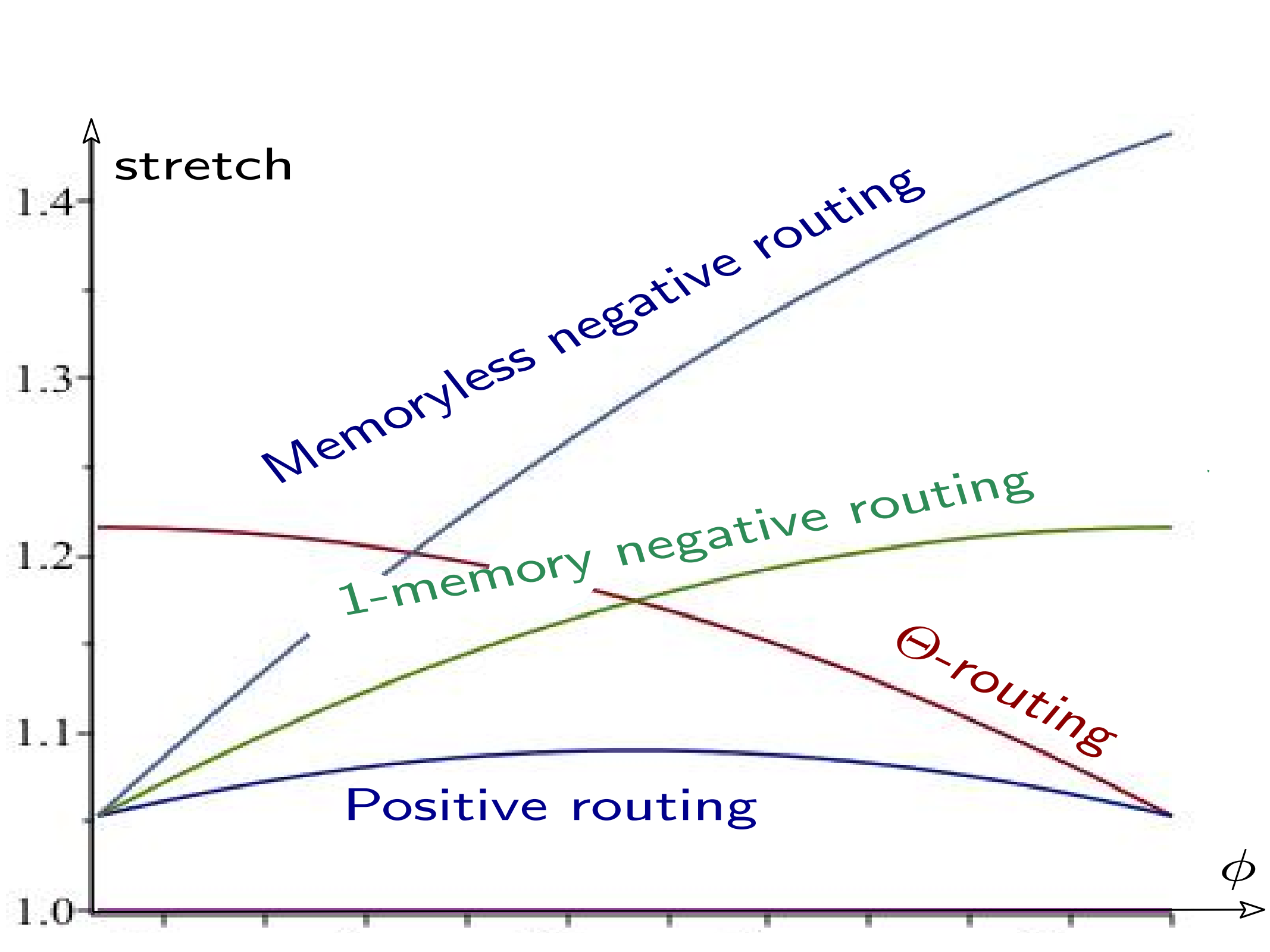
1-Memory negative routing



1-Memory negative routing

Side + Side




stretch
1.4
1.3
1.2
1.1
1.0
Memoryless negative routing
1-memory negative routing
Θ-routing
Positive routing
ϕ

Expected length (Poisson distribution)

Routing		Max on ϕ	Averaging on ϕ
Θ_6			
Positive			
Memoryless neg.			
1-Memory neg.			

Expected length (Poisson distribution)

Routing		Max on ϕ	Averaging on ϕ
Θ_6	∞		
Positive	2		
Memoryless neg.	$\frac{5}{\sqrt{3}}$		
1-Memory neg.	$\simeq 2.89$		

Worst-case

Expected length (Poisson distribution)

Routing		Max on ϕ	Averaging on ϕ
Θ_6	∞	$\frac{1}{6}(3\ln 3+4) \simeq 1.2160$	
Positive	2	$\frac{\sqrt{3}}{12}(3\ln 3+4)\left((\sqrt{3}-2)\cos\frac{5\pi}{12}-\sin\frac{5\pi}{12}\right)$ $\simeq 1.0902$	
Memoryless neg.	$\frac{5}{\sqrt{3}}$	$\frac{\sqrt{3}}{24}(3\ln 3+4)(1+\sqrt{3})$ $\simeq 1.4385$	
1-Memory neg.	$\simeq 2.89$	$\frac{1}{6}(3\ln 3+4) \simeq 1.2160$	

Worst-case

Expected length (Poisson distribution)

Routing		Max on ϕ	Averaging on ϕ
Θ_6	∞	$\frac{1}{6}(3\ln 3+4) \simeq 1.2160$	$\frac{1}{2\pi}(3\ln 3+4) \simeq 1.1612$
Positive	2	$\frac{\sqrt{3}}{12}(3\ln 3+4)\left((\sqrt{3}-2)\cos\frac{5\pi}{12}-\sin\frac{5\pi}{12}\right)$ $\simeq 1.0902$	$\frac{1}{\pi}(2\sqrt{3}-3)(3\ln 3+4)$ $\simeq 1.0778$
Memoryless neg.	$\frac{5}{\sqrt{3}}$	$\frac{\sqrt{3}}{24}(3\ln 3+4)(1+\sqrt{3})$ $\simeq 1.4385$	$\frac{3(\sqrt{3}-1)}{4\pi}(3\ln 3+4)$ $\simeq 1.2751$
1-Memory neg.	$\simeq 2.89$	$\frac{1}{6}(3\ln 3+4) \simeq 1.2160$	$\frac{1}{2\pi}(3\ln 3+4) \simeq 1.1612$

Worst-case